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Monetary policy, asset prices and model uncertainty

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Abstract: Using a macroeconomic model with asset prices, we analyze how optimal monetary policy, and macroeconomic dynamics and performance are affected by the central bank's desire to be robust against model misspecifications. Considering the worst-case model, we show that an increase in the central bank's preference for robustness requires a more aggressive reaction of the optimal nominal interest rate with respect to expected inflation and inflation shocks. According to the value of structural parameters, the economic equilibrium can be stable or saddle-point stable. In both cases, the speed of dynamic convergence is smaller under robust control compared to a benchmark case without it. Finally, an increase in the preference for robustness reinforces the reaction of current and expected future inflation, asset prices and output-gap to inflation shocks. However, the preference for robustness has no effect on the reaction of asset prices to the shocks affecting goods demand and financial markets.

Keywords: Monetary policy, asset prices, model uncertainty, macro-financial stability.

JEL Classification: E44, E52, E58

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1. Introduction

In the last decade, many central banks have succeeded to stabilise inflation at low levels in introducing the inflation-targeting framework. However, some economists doubt that this environment might have favoured large swings in asset prices.[1] According to the Bank for International Settlements (BIS, 2007), "our understanding of economic processes may even be less today than it was in the past". The reason is an increase in fundamental uncertainties about how the economy works. In fact, technological progress and globalization have transformed the production. Meanwhile, financial innovations bring new uncertainties into the financial system by introducing incessantly new products whose risks are often not well evaluated by market operators and regulators.

In this new economic environment, recent researches have developed different approaches to robust monetary policy design in order to tackle the economic and financial uncertainties. Without the possibility to have a complete description of reality, a policymaker is likely to prefer basing policy on principles that are valid also if the assumptions on which the model is founded differ from reality. In other words, policy prescriptions should be robust to reasonable deviations from the benchmark model. The growing literature on monetary policy robustness has been developed into three directions[2]. The first one leads to what has been called robustly optimal instrument rules (Svensson and Woodford, 2004; Giannoni and Woodford, 2003a, 2003b). As these instrument rules do not depend on the specification of the generating

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[1] According to Borio and Lowe (2002), low inflation can promote financial imbalances, regardless of the underlying cause of an asset price boom. For example, by generating optimism about the macroeconomic environment, low inflation might cause asset prices to rise more in response to an increase in productivity growth than they otherwise would.

processes of exogenous disturbances in the model, they are, therefore, robust to misspecification in these processes. The second one, initiated by Hansen and Sargent (2001, 2003, 2007), corresponds to robust control approach to the decision problem of agents who face model uncertainty. This approach to model uncertainty focuses exclusively on the worst-case outcome within a set of admissible models. In the sense of Hansen and Sargent, robust monetary policies are designed to perform well in worst-case scenarios, by minimizing the consequences of the worst-case specification of the policymaker's reference model. These policies arise as the equilibrium in a game between the monetary authorities and an evil agent who chooses additive model misspecification to make the authorities look as bad as possible. While these two approaches to robust policies appear quite distinct, Walsh (2004) has demonstrated that both approaches lead to exactly the same implicit optimal instrument rule for the policy maker in a standard, forward-looking, new Keynesian model. The third approach to robustness considers the structured Knightian uncertainty. It is assumed that the uncertainty is located in one or more specific parameters of the model, but the true values of these parameters are known only to be bounded between minimum and maximum conceivable values (Onatski and Stock, 2002; Giannoni, 2002, 2007; Tetlow and von zur Muehlen, 2004). What these three approaches share is a focus on the concept of *uncertainty* in the sense of Knight instead of that of *risk*.[3]

However, the literature on monetary policy robustness treats generally model uncertainties affecting the Phillips curve and the IS equation (in closed-economy models) as well as the uncovered interest rate parity (in open economy models), neglecting the role of asset prices and

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[2] Another current of research studies the robustness of a monetary policy rule across different models (Lucas-type transmission mechanism, backward-looking and forward-looking models). See e.g. Levin and Williams (2003).

[3] These approaches are more appropriate if policymakers face Knightian uncertainty and therefore be unable to assign probabilities to alternative specifications of its model. In contrast, in the Bayesian approach, central banks would take into account all possible outcomes in the specified model set and assign weights to each competing model according to its perceived probability (e.g. Batini *et al.*, 2006; Brock *et al.*, 2007).

the misspecification affecting asset pricing. In contrast, the work of Tetlow (2006) studies model uncertainty affecting only asset prices.

Meanwhile, ignoring model misspecifications, recent studies on the relationship between asset prices and monetary policy consider the benefits of allowing the monetary authority to respond to asset prices in a monetary policy rule. The essential question is not about whether the central bank's objective function should include asset prices, but how an inflation-targeting central bank can most effectively fulfill its objectives. A more general case can be made for central banks to react to asset prices in the normal course of policy making without trying to target asset prices (e.g. Cecchetti *et al.*, 2000; Cecchetti *et al.*, 2003; Filardo, 2000, 2004; Bean, 2003; Disyatat, 2005; Akram and Eitrheim, 2008; and Gilchrist and Saito, 2008). Bernanke and Gertler (1999, 2001) suggest that monetary policy should not respond to changes in asset prices, except in so far as they signal changes in expected inflation. This topic seems very crucial since responding to asset price fluctuations is likely to increase significantly macroeconomic stability only if bubbles are identified in their infancy, which is by definition the time when they are most difficult to identify. But, even if one could successfully identify bubbles, there are other reasons why a monetary authority might not react directly to asset prices since the monetary authorities might not have an exact model describing the price dynamics of financial assets. In fact, many financial prices are noisy and volatile making signal extraction difficult. Furthermore, current asset prices reflect expectations about future monetary policy and risk premium integrated in asset prices tends to vary over time.

Using the dynamic framework of Bernanke et al. (1999), Tetlow (2006) introduces parametric model uncertainty by assuming that the central bank only knows the range in which the growth rate of the stock prices lies. Tetlow has focused only on misspecifications of the

growth rate of asset prices with an application to the US economy in the presence of stock market bubbles. He has found that a direct reaction to stock prices in a policy rule reduces inflation and output volatility only marginally. However, the approach of parametric model uncertainty needs numerical simulations to appreciate the properties of the solutions.

The aim of our paper is to contribute to the literature of robust monetary policy by studying how the robust control approach of Hansen and Sargent (2007) affects the relationship between monetary policies and asset prices as well as the dynamic stability of the economic system. The choice of this approach is motivated by the fact that it allows us to obtain closed-form solutions for the optimal robust policy and the equilibrium behaviour of the economy. We can therefore discuss analytically the macroeconomic performance and dynamics of the economy in considering different model regime shifts due to modifications of exogenous parameters or model misspecifications.

We also allow, following Leitemo and Söderström (2008b), the policymaker's preference for robustness to differ across equations, reflecting the confidence the policymaker has in each relationship. Hence, we consider several different types of misspecification within the model, affecting respectively firms' price-setting, consumer behaviour and determination of asset prices. Then, it is possible to examine the effect of each particular misspecification on the robust monetary policy. The importance of the ability to focus on specification errors in particular equations is justified on the ground that policymakers are more confident in some relationships than in others, and so regard some types of specification errors to be more important than others. In the presence of asset prices, monetary policymakers are particularly uncertain about interactions between monetary policy, the asset prices and the economy. Adopting this approach,

we are able to analyze the design of monetary policy under such specific model uncertainty while keeping other potential sources of misspecification fixed.

The seminal analysis of Brainard (1967) has shown that increased uncertainty about the effects of policy should lead generally to more cautious policy behaviour. However, introducing Knightian model uncertainty, it has been shown that an increased preference for robustness tends to lead to more aggressive policy behaviour in closed economy (e.g, Onatski and Stock, 2002; Hansen and Sargent, 2001; Giannoni, 2002; Giordani and Söderlind, 2004; Leitemo and Söderström, 2008a). Leitemo and Söderström (2008b) show that this result does not carry over to the open economy where the optimal robust policy can be either more aggressive or more cautious than the non-robust policy.

We show in the present paper that the closed economy results about the robust policy are not modified by the introduction of asset prices, independently of on the source of misspecification and the type of disturbance affecting the economy. In effect, the asset prices have some similar properties than the exchange rate. However, they do not affect, as that is the case for the latter, the Phillips curve.

A second set of results concerns the effects on the macroeconomic dynamics of the central bank's preference for model robustness. This preference modifies the reaction of the nominal interest rate to the expected future inflation and asset prices and hence influences the dynamic stability of the economy. Generally, the dynamic stability of the economy is not affected but the speed of convergence to the equilibrium is reduced when the preference for robustness increases.

In the next section, we lay down a stylised macroeconomic model with asset prices. In the section after, we study the dynamic stability of the economy when monetary policy is conducted without model uncertainty. In the fourth section, we derive the optimal robust policy for the

worst-case model and examine the effect of the preference for robustness on the macroeconomic and financial stability. In the fifth section, we solve for the equilibrium solutions of endogenous variables under robust control and study their sensibilities to the preference for robustness. We summarize our findings and conclude in the last section.

2. A stylised macroeconomic model with asset prices

Our stylised system of macroeconomic equations is the following[4]:

$$\pi_t = \beta E_t \pi_{t+1} + \delta y_t + \varepsilon_t^\pi, \qquad \text{with } 0 < \beta < 1 \text{ and } \delta > 0, \tag{1}$$

$$y_t = E_t y_{t+1} - \alpha_1 (i_t - E_t \pi_{t+1}) + \alpha_2 A_t + \varepsilon_t^d, \qquad \text{with } \alpha_1, \alpha_2 > 0, \tag{2}$$

$$A_t = \gamma_1 (y_t - \varepsilon_t^\pi) + \gamma_2 E_t A_{t+1} - (i_t - E_t \pi_{t+1}) + \varepsilon_t^e, \qquad \text{with } \gamma_1, \gamma_2 > 0, \tag{3}$$

$$\varepsilon_{t+1}^j = \rho_j \varepsilon_t^j + e_{t+1}^j, \qquad \text{with } j = \pi, d, e \; ; \; 0 \le \rho_j \le 1 \text{ and } E_t e_{t+1}^j = 0 \; ; \tag{4}$$

where π_t denotes the inflation rate, $E_t \pi_{t+1}$ the private sector's expectation of future inflation with E_t as the expectation operator reflecting the hypothesis of rational expectations, y_t the output gap, A_t the equity price, $E_t A_{t+1}$ the expected equity price, i_t the nominal interest rate, ε_t^π an inflation or cost shock, ε_t^d the demand shock and ε_t^e a disturbance which represents an equity premium shock of the type discussed in Cecchetti *et al.* (2000).

Equation (1) is a forward-looking Phillips curve based on optimizing private sector behaviour and nominal rigidities that has been used extensively in the recent literature on monetary policy (Clarida *et al.*, 1999). The parameter β represents the private discount factor

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[4] A similar model is used by Alexandre and Bação (2005) except for their hybrid Phillips curve.

which is positive but inferior to unity. The parameter δ is the output gap elasticity of inflation and captures the effects of the gap on real marginal costs and marginal cost on inflation.

Equation (2) is the aggregate demand (or IS) equation and, except for the asset price term, can be derived from a dynamic general equilibrium model with optimising agents (McCallum and Nelson, 1999). As in Alexandre and Bação (2005), an *ad hoc* term ($\alpha_2 A_t$) is added to incorporate a wealth effect[5] and which can be justified on the ground that it is a shortcut within the spirit of the debate about the role of asset prices in the transmission mechanism of monetary policy (see, e.g., Cecchetti *et al.*, 2000).

Equation (3) is derived from a standard dividend model of asset pricing. Fundamental real equity prices are a function of next-period dividends assumed to depend positively on current output and the supply shock, expected future dividends (incorporated into the expected equity price), and negatively on the real interest rate. This is supported by the majority of empirical studies examining the effect of macroeconomic variables on the stock market[6].

Finally, equation (4) defines the shocks in the system as first-order autoregressive processes where ρ_j represents the degree of persistence.

3. Design of monetary policy without model uncertainty

Consider a benchmark case where the central bank knows exactly the true structure of the economy. Monetary policy is then implemented to minimize the conditional expectation of the central bank's loss function:

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[5] Disyatat (2005), Kontonikas and Ioannidis (2005), Akram and Eitrheim (2006), and Kontonikas and Montagnoli (2006) have also introduced similarly wealth effect on aggregate demand and have formulated dynamic equations of asset prices which are different from the one used in this model.

$$L = \frac{1}{2} E_t [\lambda (\pi_t - \pi^T)^2 + (y_t - y^*)^2]. \tag{5}$$

Where λ denotes the relative weight that the central bank assigns to the inflation target, the inflation target π^T and y^* the output gap target.

The first-order condition is:

$$\frac{\partial L}{\partial \pi_t} = E_t [\lambda (\pi_t - \pi^T) + (y_t - y^*) \frac{\partial y_t}{\partial \pi_t}] = 0. \tag{6}$$

Deriving equation (1) to find $\dfrac{\partial y_t}{\partial \pi_t}$ and substituting it in the condition (6), it follows:

$$\lambda (\pi_t - \pi^T) + (y_t - y^*) \frac{1}{\delta} = 0. \tag{7}$$

Using equations (1), (2) and (7) to eliminate π_t and y_t yields the optimal nominal interest rate rule[7]:

$$i_t = E_t \pi_{t+1} + \frac{\delta \lambda}{\alpha_1 (1 + \lambda \delta^2)} (\beta E_t \pi_{t+1} + \delta y^* - \pi^T + \varepsilon_t^\pi) + \frac{1}{\alpha_1} (E_t y_{t+1} - y^*) + \frac{\alpha_2}{\alpha_1} A_t + \frac{\varepsilon_t^d}{\alpha_1}. \tag{8}$$

According to equation (8), the nominal interest rate is raised when there is an increase in the inflation expectations, in the expected output gap, in the asset prices as well as when there is a positive demand shock or a negative inflation shock. A reaction to current asset prices allows the monetary authorities to indirectly take account of current output, but also the expected asset prices (that a central banker is generally reluctant to predict) and shocks affecting asset pricing.

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[6] See, among others, Fama (1981), Conover, Jensen and Johnson (1999).

[7] An alternative interest rate rule is to determine the optimal interest rate only in terms of expected future variables and exogenous shocks. However, central banks might be reluctant to base their interest rate decision on expected future asset prices.

In order to study the macro-financial stability of the economy, we construct a dynamic system with two endogenous variables, i.e. inflation and asset prices. Taking conditional expectations of equation (7) for period $t+1$ leads to

$$E_t y_{t+1} - y^* = -\lambda\delta(E_t\pi_{t+1} - \pi^T).\tag{9}$$

Using equations (1) and (7) to eliminate y_t, we obtain:

$$E_t\pi_{t+1} = \frac{(1+\lambda\delta^2)}{\beta}\pi_t - \frac{\delta}{\beta}y^* - \frac{\lambda\delta^2}{\beta}\pi^T - \frac{1}{\beta}\varepsilon_t^\pi\tag{10}$$

Manipulating equations (3), (7), (8), (9) and (10) to remove i_t, $E_t y_{t+1}$ and $E_t\pi_{t+1}$ leads to:

$$E_t A_{t+1} = \frac{\alpha_1+\alpha_2}{\gamma_2\alpha_1}A_t + \frac{\gamma_1\alpha_1\beta+\beta-(1+\lambda\delta^2)}{\gamma_2\alpha_1\beta}\delta\pi_t - \frac{\gamma_1\alpha_1\beta-\lambda\delta^2}{\gamma_2\alpha_1\beta}(\delta\pi^T + y^*) - \frac{1}{\gamma_2}\varepsilon_t^e + \frac{\gamma_1\alpha_1\beta+\delta}{\gamma_2\alpha_1\beta}\varepsilon_t^\pi + \frac{1}{\gamma_2\alpha_1}\varepsilon_t^d.\tag{11}$$

Rearrange then equations (10) and (11) in matrix form as follows:

$$\begin{bmatrix} E_t\pi_{t+1} \\ E_t A_{t+1} \end{bmatrix} = \begin{bmatrix} \dfrac{(1+\lambda\delta^2)}{\beta} & 0 \\ \dfrac{\gamma_1\alpha_1\beta+\beta-(1+\lambda\delta^2)}{\gamma_2\alpha_1\beta}\delta & \dfrac{\alpha_1+\alpha_2}{\gamma_2\alpha_1} \end{bmatrix}\begin{bmatrix} \pi_t \\ A_t \end{bmatrix} + \begin{bmatrix} -\dfrac{\delta}{\beta}y^* - \dfrac{\lambda\delta^2}{\beta}\pi^T - \dfrac{1}{\beta}\varepsilon_t^\pi \\ -\dfrac{\gamma_1\alpha_1\beta-\lambda\delta^2}{\gamma_2\alpha_1\beta}(\delta\pi^T + y^*) - \dfrac{1}{\gamma_2}\varepsilon_t^e + \dfrac{\gamma_1\alpha_1\beta+\delta}{\gamma_2\alpha_1\beta}\varepsilon_t^\pi + \dfrac{1}{\gamma_2\alpha_1}\varepsilon_t^d \end{bmatrix}.\tag{12}$$

Proposition 1. *Under inflation targeting regime, the dynamic system (12) has a stable equilibrium for $\gamma_2 < \frac{\alpha_1+\alpha_2}{\alpha_1}$. A saddle-point stable equilibrium emerges for $\gamma_2 > \frac{\alpha_1+\alpha_2}{\alpha_1}$.*

Proof. The stability matrix of the dynamic system (12) has two eigenvalues $E_1 = \frac{1+\lambda\delta^2}{\beta}$ and $E_2 = \frac{\alpha_1+\alpha_2}{\gamma_2\alpha_1}$. If $\gamma_2 < \frac{\alpha_1+\alpha_2}{\alpha_1}$, their values are superior to unity. That ensures, when the system is perturbed, the convergence to the equilibrium whatever the nature of these dynamic variables (predetermined or non-predetermined). If $\gamma_2 > \frac{\alpha_1+\alpha_2}{\alpha_1}$, then the second eigenvalue will be inferior

to unity. In this case, the system will have one stable and one instable eigenvalue. It will be characterized by a saddle-point stable equilibrium under the assumption that asset prices are a non-predetermined variable and expected inflation rate a predetermined one. *Q.E.D.*

The assumption introduced above can be justified on the ground that asset prices adjust more quickly than expected inflation in a low inflation environment. Asset prices, particularly these quoted in continuous time on a centralized market, are much more flexible than goods prices and wages. They are free to make discrete instantaneous jumps in response to "news" concerning all previously unanticipated current or future changes in exogenous variables and policy instruments. Therefore, the asset prices, A_t, clearing an efficient financial market, are considered as a non-predetermined variable. On the other hand, in a low inflation environment, inflation rate and hence expected inflation (π_t and $E_t \pi_{t+1}$), resulting from a relatively slow adjustment of goods prices and wages due to different factors (such as menu costs, overlapping contracts or partial adjustment), are considered as a predetermined variable[8]. This distinction is essentially based on the relative speed of adjustment of these two variables.

4. Monetary policy under model uncertainty

Although the central bank perceives the benchmark model described by equations (1), (2) and (3) as the most likely specification, it realizes that the true model may deviate from the benchmark without, however, being able to specify a probability distribution for deviations. To

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[8] See e.g. Buiter and Panigirtzoglou (2003) for a similar assumption concerning inflation rate. In practice, if all price and wage contracts are short term and/or if inflation is high (i.e. superior to the cost of price and wage adjustment for

take account of such misspecifications, we introduce in equations (1), (2) and (3) a second type of disturbances, respectively denoted by h_t, w_t and u_t. The disturbances are controlled, in the sense of Hansen and Sargent (2007), by a fictitious "evil agent" representing the policymaker's worst fears concerning specification errors. Thus, the model with misspecifications is given by

$$\pi_t = \beta E_t \pi_{t+1} + \delta y_t + \varepsilon_t^\pi + h_t, \qquad \text{with } 0 < \beta < 1, \ \delta > 0, \tag{13}$$

$$y_t = E_t y_{t+1} - \alpha_1(i_t - E_t \pi_{t+1}) + \alpha_2 A_t + \varepsilon_t^d + w_t, \qquad \text{with } \alpha_1, \alpha_2 > 0, \tag{14}$$

$$A_t = \gamma_1(y_t - \varepsilon_t^\pi) + \gamma_2 E_t A_{t+1} - (i_t - E_t \pi_{t+1}) + \varepsilon_t^e + u_t, \qquad \text{with } \gamma_1, \gamma_2 > 0 \tag{15}$$

As it is common in the robust control literature, we assume that the central bank allocates a budget χ^2 to the evil agent who creates misspecifications under the following budget constraints:

$$E_t \sum_{n=0}^{\infty} \rho^n h_{t+n}^2 \leq \chi^2, \tag{16}$$

$$E_t \sum_{n=0}^{\infty} \rho^n w_{t+n}^2 \leq \chi^2, \tag{17}$$

$$E_t \sum_{n=0}^{\infty} \rho^n u_{t+n}^2 \leq \chi^2. \tag{18}$$

To design the robust monetary policy, the central bank takes into account a certain degree of model misspecifications by minimizing its objective function in the worst possible model within a given set of plausible models. Monetary policy is implemented to minimize the conditional expectation of the loss function. The robust monetary policy is obtained by solving the min-max problem

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all firms and trade union) and they reacts in a forward-looking manner, we can also consider current and expected inflation as jumping variable.

$$\min_{\{i_t\}} \max_{\{h_t, w_t, u_t\}} E_t \sum_0^\infty \frac{1}{2} [\lambda V(\pi_t) + V(y_t) - \theta_h h_t^2 - \theta_w w_t^2 - \theta_u u_t^2], \tag{19}$$

subject to the misspecified model (14)–(15) and the evil agent's budget constraints (16)–(18). The central bank sets the nominal interest rate to minimize the value of its intertemporal loss function, whereas the evil agent creates misspecifications to maximize the central bank's loss given its budget constraints. The parameters θ_i, $i = h, w, u$, determine the set of models available to the evil agent that the policymaker wants to be robust against. They represent the central bank preference for robustness: the higher the value θ_i, the lower is the preference for robustness. They are related to the evil agent's budget: as $\chi^2 \to 0$, $\theta_i \to \infty$, and the model specification errors approach zero.

The Lagrangian for this problem is given by

$$L = \sum_0^\infty \begin{Bmatrix} \frac{1}{2} E_t [\lambda(\pi_t - \pi^T)^2 + (y_t - y^*)^2 - \theta_h h_t^2 - \theta_w w_t^2 - \theta_u u_t^2] + \\ -\mu_t^\pi [\pi_t - \beta E_t \pi_{t+1} - \delta y_t - \varepsilon_t^\pi - h_t] \\ -\mu_t^y [y_t - E_t y_{t+1} + \alpha_1 (i_t - E_t \pi_{t+1}) - \alpha_2 A_t - \varepsilon_t^d - w_t] \\ -\mu_t^A [A_t - \gamma_1 (y_t - \varepsilon_t^\pi) - \gamma_2 E_t A_{t+1} + (i_t - E_t \pi_{t+1}) - \varepsilon_t^e - u_t] \end{Bmatrix},$$

where μ_t^j, $j = \pi, y, A$, are Lagrange multipliers on the constraints (13), (14) and (15) respectively. The first-order conditions for the min-max problem are:

$$\frac{\partial L}{\partial \pi_t} = \lambda(\pi_t - \pi^T) - \mu_t^\pi = 0, \tag{20}$$

$$\frac{\partial L}{\partial y_t} = (y_t - y^*) - \mu_t^y + \delta\mu_t^\pi + \gamma_1\mu_t^A = 0, \tag{21}$$

$$\frac{\partial L}{\partial i_t} = -\alpha_1\mu_t^y - \mu_t^A = 0, \tag{22}$$

$$\frac{\partial L}{\partial A_t} = \alpha_2 \mu_t^y - \mu_t^A = 0, \tag{23}$$

$$\frac{\partial L}{\partial h_t} = -\theta_h h_t + \mu_t^\pi = 0, \tag{24}$$

$$\frac{\partial L}{\partial w_t} = -\theta_w w_t + \mu_t^y = 0, \tag{25}$$

$$\frac{\partial L}{\partial u_t} = -\theta_u u_t + \mu_t^A = 0. \tag{26}$$

From conditions (22) and (23), it follows that, $\mu_t^y = \mu_t^A = 0$. Using this result and conditions (25) and (26), we obtain:

$$w_t = 0, \tag{27}$$

$$u_t = 0. \tag{28}$$

The optimality conditions (20) and (21) imply that the preference for robustness does not influence the optimal trade-off between the inflation and the output gap. The solution of w_t given by equation (27) shows that the optimal misspecification in the *IS* equation is always zero as the central bank is able to neutralize misspecification in the output equation by an appropriate adjustment of the interest rate. In fact, as discussed by Leitemo and Soderstrom, (2008a, b), the central bank does not fear such misspecification because its loss function is not affected by these interest rate movements. The same explanation holds for the solution of u_t given by equation (28), meaning that the optimal misspecification in the asset pricing equation (15) is always zero. We remark that, in Leitemo and Soderstrom, the exchange rate equation is prone to misspecification. Even if asset prices have similar impact on output equation as exchange rate, the asset pricing equation in our model is not affected by misspecification. The explanation is

that the asset prices do not affect directly the Phillips curve. Meanwhile, the latter is influenced by the exchange rate in the model of Leitemo and Soderstrom.

Taking account of the results $\mu_t^y = \mu_t^A = 0$, $u_t = 0$ and $w_t = 0$, the first-order conditions (20), (21) and (24) allow to obtain:

$$y_t - y^* = -\lambda\delta(\pi_t - \pi^T), \tag{29}$$

$$y_t - y^* = -\delta\theta_h h_t. \tag{30}$$

The second-order conditions, with regard to model misspecifications, of the central bank's min-max problem are obtained in deriving the first-order conditions (24)-(26), taking account of the first-order condition (21), the results $\mu_t^y = \mu_t^A = 0$ and equation (30):

$$\frac{\partial^2 L}{\partial h_t^2} = E[\frac{1}{\delta^2} - \theta_h] < 0 \Rightarrow \theta_h > \frac{1}{\delta^2}, \tag{31}$$

$$\frac{\partial^2 L}{\partial w_t^2} = 0, \tag{32}$$

$$\frac{\partial^2 L}{\partial u_t^2} = 0. \tag{33}$$

Using equations (13), (29) and (30), we get:

$$\pi_t = \frac{\theta_h}{\theta_h - \lambda + \lambda\delta^2\theta_h}(\beta E_t\pi_{t+1} + \delta y^* - \pi^T + \varepsilon_t^\pi) + \pi^T, \tag{34}$$

$$y_t = y^* - \frac{\lambda\delta\theta_h}{\theta_h - \lambda + \lambda\delta^2\theta_h}(\beta E_t\pi_{t+1} + \delta y^* - \pi^T + \varepsilon_t^\pi), \tag{35}$$

$$h_t = \frac{\lambda}{\theta_h - \lambda + \lambda\delta^2\theta_h}(\beta E_t\pi_{t+1} + \delta y^* - \pi^T + \varepsilon_t^\pi). \tag{36}$$

Substituting w_t, π_t and y_t, given by equations (27), (34) and (35) respectively, into equation (13), we obtain the nominal interest rate rule for the worst-case model as follows:

$$i_t = E_t \pi_{t+1} + \frac{\lambda \delta \theta_h}{\alpha_1(\theta_h - \lambda + \theta_h \lambda \delta^2)}(\beta E_t \pi_{t+1} + \delta y^* + \varepsilon_t^\pi - \pi^T) + \frac{1}{\alpha_1}(E_t y_{t+1} - y^*) + \frac{\alpha_2}{\alpha_1} A_t + \frac{1}{\alpha_1} \varepsilon_t^d . \quad (37)$$

Proposition 2. *The reaction of the optimal nominal interest rate to the expected inflation and inflation shocks is increasing with the preference for robustness against output misspecification.*

Proof. Deriving the nominal interest rate determined by the rule (37) with respect to expected inflation and inflation shocks respectively, and then deriving the resulting partial derivatives with regard to the preference for robustness leads to

$$\frac{\partial^2 i_t}{\partial E_t \pi_{t+1} \partial \theta_h} = \frac{\beta \partial^2 i_t}{\partial \varepsilon_t^\pi \partial \theta_h} = -\frac{\beta \delta \lambda^2}{\alpha_1(\theta_h - \lambda + \lambda \theta_h \delta^2)^2} < 0 . \quad (38)$$

In fact, a higher preference for robustness, which corresponds to a lower θ_h, implies a stronger reaction of the optimal nominal interest rate relative to the expected inflation. In other words, an increase in output misspecification is equivalent to a positive inflation shock. *Q.E.D.*

Proposition 3. *The preference for robustness does not change the stability propriety of the dynamic system, i.e. the equilibrium is stable if $\gamma_2 < \frac{\alpha_1 + \alpha_2}{\alpha_1}$ or saddle-point stable if $\gamma_2 > \frac{\alpha_1 + \alpha_2}{\alpha_1}$. The only modification introduced by the optimal robust monetary policy is that the speed of dynamic convergence to the equilibrium is smaller under robust control compared to the benchmark case.*

Proof. The difference equation of expected inflation can be directly obtained from equation (34) and that of asset prices can be reformulated using equations (13)-(15), (27)-(29) and (36), to

eliminate other endogenous variables, i.e. i_t, y_t, $E_t \pi_{t+1}$ and $E_t y_{t+1}$ (Appendix A). The system of

dynamic equations is presented in matrix form as:

$$
\begin{bmatrix} E_t \pi_{t+1} \\ E_t A_{t+1} \end{bmatrix} = \begin{bmatrix} \dfrac{\theta_h - \lambda + \lambda \delta^2 \theta_h}{\theta_h \beta} & 0 \\ \dfrac{\gamma_1 \lambda \delta}{\gamma_2} - \dfrac{\lambda \delta (1 - \beta + \lambda \delta^2)(\theta_h + \lambda \delta^2 \theta_h) + \lambda^2 \delta(1 + \lambda \delta^2)}{\alpha_1 \gamma_2 \beta (\theta_h + \lambda \delta^2 \theta_h)} & \dfrac{\alpha_1 + \alpha_2}{\gamma_2 \alpha_1} \end{bmatrix} \begin{bmatrix} \pi_t \\ A_t \end{bmatrix} +
$$

$$
\begin{bmatrix} -\dfrac{\theta_h(1-\beta) - \lambda + \lambda \delta^2 \theta_h}{\theta_h \beta} \pi^T - \delta y^* - \varepsilon_t^\pi \\ +(\dfrac{\lambda \delta^2}{\alpha_1 \beta \gamma_2} - \dfrac{\gamma_1}{\gamma_2})y^* + \left[\dfrac{\lambda^2 \delta[\delta^2(\theta_h - \lambda + \lambda \delta^2 \theta_h) - 1]}{\alpha_1 \beta \gamma_2 (\theta_h + \lambda \delta^2 \theta_h)} - \dfrac{\gamma_1 \lambda \delta}{\gamma_2}\right] \pi^T + (\dfrac{\lambda \delta}{\alpha_1 \beta \gamma_2} + \dfrac{\gamma_1}{\gamma_2})\varepsilon_t^\pi + \dfrac{\varepsilon_t^d}{\alpha_1 \gamma_2} - \dfrac{\varepsilon_t^e}{\gamma_2} \end{bmatrix}.
$$

$$(39)$$

Under the inflation-targeting regime with robust control, the stability matrix of (39) has two

eigenvalues: $E_1^r = \frac{\theta_h - \lambda + \lambda \delta^2 \theta_h}{\theta_h \beta}$ and $E_2^r = \frac{\alpha_1 + \alpha_2}{\gamma_2 \alpha_1}$. According to the second-order condition (31), the

first eigenvalue is always superior to unity:

$$
E_1^r = \frac{\theta_h - \lambda + \lambda \delta^2 \theta_h}{\theta_h \beta} = \frac{\theta_h + \lambda(\delta^2 \theta_h - 1)}{\theta_h \beta} > 1.
$$

If $\gamma_2 < \frac{\alpha_1 + \alpha_2}{\alpha_1}$, the second eigenvalue is also superior to unity, i.e. $E_2^r = \frac{\alpha_1 + \alpha_2}{\gamma_2 \alpha_1} > 1$. With two

eigenvalues superior to unity, the equilibrium is stable.

If $\gamma_2 > \frac{\alpha_1 + \alpha_2}{\alpha_1}$, the system has one stable eigenvalue and one unstable eigenvalue and

consequently a saddle-point stable equilibrium, given that the inflation rate is assumed to be a

predetermined variable and the asset prices a non-predetermined one.

The second part of the proposition 3 can be proved in comparing E_1^r and E_1. In effect, it is

straightforward to show that the value of the eigenvalue E_1^r is smaller relative to E_1 :

$$
E_1^r = \frac{1 + \lambda \delta^2}{\beta} - \frac{\lambda}{\theta_h \beta} < E_1 = \frac{1 + \lambda \delta^2}{\beta}, \qquad \forall \theta_h > 0. \tag{40}
$$

Since the second eigenvalue is the same under the two monetary policy regimes, i.e.

$E_2 = E_2^r = \frac{\alpha_1 + \alpha_2}{\gamma_2 \alpha_1}$, the speed of convergence of inflation and asset prices to their equilibrium

values is reduced whenever the central bank has higher preference for robustness (lower value of

θ_h). Q.E.D.

5. Model uncertainty and macroeconomic performance

Using the method of indeterminate coefficients (McCallum, 1983) to solve the decomposable

dynamic system (39), we obtain the equilibrium solutions of π_t, $E_t\pi_{t+1}$, A_t and $E_t A_{t+1}$ for the

worst-case model as follows (Appendix B):

$$\pi_t = \frac{-\lambda + \theta_h \lambda \delta^2}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} \pi^T + \frac{\theta_h \delta}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} y^* + \frac{\theta_h}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s} \varepsilon_t^\pi, \qquad (41)$$

$$E_t\pi_{t+1} = \frac{-\lambda + \theta_h \lambda \delta^2}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} \pi^T + \frac{\theta_h \delta}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} y^* + \frac{\theta_h \rho_s}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s} \varepsilon_t^\pi, \quad (42)$$

$$A_t = \frac{(\alpha_1 + \alpha_2)\{\gamma_1 \lambda \delta \theta_h (1-\beta)\pi^T + \gamma_1 [\theta_h (1-\beta) - \lambda] y^*\}}{(\alpha_1 + \alpha_2 - \alpha_1 \gamma_2)[\theta_h (1-\beta) - \lambda + \theta_h \lambda \delta^2]} - \frac{\alpha_1 + \alpha_2}{\alpha_1 (\alpha_1 + \alpha_2 - \alpha_1 \gamma_2 \rho_d)} \varepsilon_t^d +$$
$$\frac{\alpha_1 + \alpha_2}{\alpha_1 + \alpha_2 - \alpha_1 \gamma_2 \rho_e} \varepsilon_t^e - \frac{(\alpha_1 + \alpha_2)[\lambda \delta \theta_h + \gamma_1 \alpha_1 (\theta_h - \lambda + \theta_h \lambda \delta + \theta_h \lambda \delta^2) - (\gamma_1 \alpha_1 \beta + \lambda \delta)\theta_h \rho_s]}{\alpha_1 (\alpha_1 + \alpha_2 - \alpha_1 \gamma_2 \rho_s)[\theta_h (1-\beta\rho_s) - \lambda + \theta_h \lambda \delta^2]} \varepsilon_t^\pi, \qquad (43)$$

$$E_t A_{t+1} = \frac{\gamma_1 \lambda \delta \theta_1 (\alpha_1 + \alpha_2)(1-\beta)\pi^T + \gamma_1 (\alpha_1 + \alpha_2)[\theta_1 (1-\beta) - \lambda] y^*}{(\alpha_1 + \alpha_2 - \alpha_1 \gamma_2)(\theta_1 - \lambda + \theta_1 \lambda \delta^2 - \theta_1 \beta)} - \frac{(\alpha_1 + \alpha_2)\rho_d}{\alpha_1 (\alpha_1 + \alpha_2 - \alpha_1 \gamma_2 \rho_d)} \varepsilon_t^d +$$
$$\frac{(\alpha_1 + \alpha_2)\rho_e}{\alpha_1 + \alpha_2 - \alpha_1 \gamma_2 \rho_e} \varepsilon_t^e - \frac{(\alpha_1 + \alpha_2)[\lambda \delta \theta_h + \gamma_1 \alpha_1 (\theta_h - \lambda + \theta_h \lambda \delta + \theta_h \lambda \delta^2) - (\gamma_1 \alpha_1 \beta + \lambda \delta)\theta_h \rho_s]}{\alpha_1 (\alpha_1 + \alpha_2 - \alpha_1 \gamma_2 \rho_s)[\theta_h (1-\beta\rho_s) - \lambda + \theta_h \lambda \delta^2]} \rho_s \varepsilon_t^\pi, \qquad (44)$$

$$y_t = \frac{(1-\beta)\lambda \delta \theta_h}{(\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta)} \pi^T + \frac{\theta_h (1-\beta) - \lambda}{(\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta)} y^* - \frac{\lambda \delta \theta_h}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s} \varepsilon_t^\pi. \qquad (45)$$

$$E_t y_{t+1} = \frac{(1-\beta)\lambda\delta\theta_h}{\theta_h - \lambda + \theta_h\lambda\delta^2 - \theta_h\beta}\pi^T + \frac{\theta_h(1-\beta)-\lambda}{\theta_h - \lambda + \theta_h\lambda\delta^2 - \theta_h\beta}y^* - \frac{\lambda\delta\theta_h\rho_s}{\theta_h - \lambda + \theta_h\lambda\delta^2 - \theta_h\beta\rho_s}\varepsilon_t^\pi. \qquad (46)$$

The solutions given above show that only asset prices are influenced by shocks affecting goods demand and financial markets. Current and expected future inflation and output gap are only affected by the inflation shocks as the optimal monetary policy has neutralized the effects of other shocks on these variables. The preference for robustness modifies the equilibrium value of endogenous variables and therefore their reactions to inflation shocks. Its effects are summarised in the following proposition.

Porposition 4. *An increase in the preference for robustness strengthens the reaction of π_t,*

$E_t\pi_{t+1}$, A_t, $E_t A_{t+1}$ and y_t to the inflation shocks. The preference for robustness has no effect on the reaction of asset prices to the shocks affecting goods demand and financial markets.

Proof. Deriving twice the solutions of endogenous variables given by equations (41)-(46) relative to different shocks and the parameter representing the preference for robustness yields:

$$\frac{d^2\pi_t}{d\varepsilon_t^\pi d\theta_h} = \frac{-\lambda}{(\theta_h - \lambda + \theta_h\lambda\delta^2 - \theta_h\beta\rho_s)^2} < 0, \qquad (47)$$

$$\frac{d^2 E_t\pi_{t+1}}{d\varepsilon_t^\pi d\theta_h} = \frac{-\rho_s\lambda}{(\theta_h - \lambda + \theta_h\lambda\delta^2 - \theta_h\beta\rho_s)^2} < 0, \qquad (48)$$

$$\frac{d^2 A_t}{d\varepsilon_t^\pi d\theta_h} = \frac{(\alpha_1 + \alpha_2)\lambda^2[\delta(1-\rho_s) + \gamma_1\alpha_1\delta]}{\alpha_1(\alpha_1 + \alpha_2 - \alpha_1\gamma_2\rho_s)[\theta_h(1-\beta\rho_s) - \lambda + \theta_h\lambda\delta^2]^2} > 0, \qquad (49)$$

$$\frac{d^2 E_t A_{t+1}}{d\varepsilon_t^\pi d\theta_h} = \frac{\rho_s(\alpha_1 + \alpha_2)\lambda^2[\delta(1-\rho_s) + \gamma_1\alpha_1\delta]}{\alpha_1(\alpha_1 + \alpha_2 - \alpha_1\gamma_2\rho_s)[\theta_h(1-\beta\rho_s) - \lambda + \theta_h\lambda\delta^2]^2} > 0, \qquad (50)$$

$$\frac{d^2 y_t}{d\varepsilon_t^\pi d\theta_h} = \frac{\lambda^2 \delta}{(\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s)^2} > 0, \tag{51}$$

$$\frac{d^2 E_t y_{t+1}}{d\varepsilon_t^\pi d\theta_h} = \frac{\lambda^2 \delta \rho_s}{(\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s)^2} > 0, \tag{52}$$

$$\frac{d^2 A_t}{d\varepsilon_t^d d\theta_h} = 0, \quad \frac{d^2 A_t}{d\varepsilon_t^e d\theta_h} = 0, \quad \frac{d^2 E_t A_{t+1}}{d\varepsilon_t^d d\theta_h} = 0, \quad \frac{d^2 E_t A_{t+1}}{d\varepsilon_t^e d\theta_h} = 0. \tag{53}$$

Q.E.D.

The above derivatives correspond to the impact of a decrease of the preference for robustness (i.e higher value for θ_h) on the effects of the exogenous shocks on the endogenous variables. For example, according to the sign of $\frac{d^2 \pi_t}{d\varepsilon_t^\pi d\theta_h}$ given by (47), a positive inflation shock will increase the inflation rate and this effect will be reinforced by an increase in the preference for robustness.

The central bank does not fear misspecification in the IS and asset pricing equations. However, the central bank could fear misspecification in the inflation equation and would respond more aggressively to inflation shocks in the worst case model. Consequently, an increase in the preference for robustness reinforces the reaction of current and expected future inflation, asset prices and output-gap to inflation shocks. Depending positively on real output, asset prices will be similarly influenced by an increase in model misspecification. As we have discussed before, the central bank does not fear misspecifications affecting the output and asset pricing equations. Therefore, the effects of shocks affecting these two equations are not associated with the preference for robustness. Furthermore, that the effects of the preference for robustness on the expected future values of endogenous variables are associated with the persistence nature of inflation shock.

6. Concluding remarks

Using a macroeconomic model with asset prices, we have analyzed how the optimal monetary policy and the dynamic behaviour of the economy are affected by the central bank's desire to be robust against model misspecification. Considering the central bank's worst-case model, we have solved analytically for the optimal robust policy and examined its dynamic implications as well as its effects on macroeconomic performance.

In this model, we have shown that the central bank is more confident about the IS and asset pricing equations and thus limits the evil agent's choice of misspecification. However, perceiving the inflation equation as being particularly prone to specification errors, the central bank allows the evil agent to introduce misspecification. An increase in the central bank's preference for robustness has unambiguous effects on the optimal monetary policy. The reaction of the optimal nominal interest rate becomes more sensitive to the expected inflation and inflation shocks, but remains unchanged in the presence of shocks affecting goods and financial markets.

Considering that the model is subject to persistent shocks, we have found that its dynamic properties are modified by the introduction of robust control independently of the dynamic stability nature of the equilibrium, i.e. stable or saddle-point stable. The speed of dynamic convergence is smaller under robust control compared to the benchmark case without robust control.

In terms of macroeconomic performance, an increase in the preference for robustness reinforces the reaction of current and expected future inflation, asset prices and output gap to

inflation shocks. Furthermore, the preference for robustness has no effect on the reaction of asset prices to shocks affecting the demand for goods and financial markets.

Appendix A. Difference equation of asset prices

Taking conditional expectations of equation (29) leads to:

$$E_t y_{t+1} = y^* - \lambda\delta(E_t\pi_{t+1} - \pi^T).$$

(A.1)

Using then equations (27), (29), (36) and (A.1) to eliminate y_t and $E_t y_{t+1}$, in equations (13) and (14), we get:

$$\pi_t = \beta E_t\pi_{t+1} + \delta y^* - \lambda\delta^2(\pi_t - \pi^T) + \varepsilon_t^\pi + \frac{\lambda}{\theta_h - \lambda + \lambda\delta^2\theta_h}(\beta E_t\pi_{t+1} + \delta y^* - \pi^T + \varepsilon_t^\pi).$$

(A.2)

$$-\lambda\delta\pi_t = -\lambda\delta E_t\pi_{t+1} - \alpha_1(i_t - E_t\pi_{t+1}) + \alpha_2 A_t + \varepsilon_t^d,$$

(A.3)

Equations (A.2) and (A.3) allow us to obtain:

$$(i_t - E_t\pi_{t+1}) = -\frac{\lambda\delta(1 - \beta + \lambda\delta^2)(\theta_h + \lambda\delta^2\theta_h) + \lambda^2\delta(1 + \lambda\delta^2)}{\alpha_1\beta(\theta_h + \lambda\delta^2\theta_h)}\pi_t + \frac{\alpha_2}{\alpha_1}A_t + \frac{\varepsilon_t^d}{\alpha_1} + \frac{\lambda\delta^2}{\alpha_1\beta}y^*$$
$$+ \frac{\lambda^2\delta}{\alpha_1\beta}\left[\frac{\delta^2(\theta_h - \lambda + \lambda\delta^2\theta_h) - 1}{\theta_h + \lambda\delta^2\theta_h}\right]\pi^T + \frac{\lambda\delta}{\alpha_1\beta}\varepsilon_t^\pi.$$

(A.4)

Using equations (28), (29) and (A.4) to eliminate u_t y_t and $E_t y_{t+1}$ and $(i_t - E_t\pi_{t+1})$ in equation (15) and rearranging the terms yield:

$$E_t A_{t+1} = \frac{\alpha_1 + \alpha_2}{\alpha_1\gamma_2}A_t + \left[\frac{\gamma_1\lambda\delta}{\gamma_2} - \frac{\lambda\delta(1 - \beta + \lambda\delta^2)(\theta_h + \lambda\delta^2\theta_h) + \lambda^2\delta(1 + \lambda\delta^2)}{\alpha_1\gamma_2\beta(\theta_h + \lambda\delta^2\theta_h)}\right]\pi_t + (\frac{\lambda\delta^2}{\alpha_1\beta\gamma_2} - \frac{\gamma_1}{\gamma_2})y^*$$
$$+ \left[\frac{\lambda^2\delta[\delta^2(\theta_h - \lambda + \lambda\delta^2\theta_h) - 1]}{\alpha_1\beta\gamma_2(\theta_h + \lambda\delta^2\theta_h)} - \frac{\gamma_1\lambda\delta}{\gamma_2}\right]\pi^T + (\frac{\lambda\delta}{\alpha_1\beta\gamma_2} + \frac{\gamma_1}{\gamma_2})\varepsilon_t^\pi + \frac{\varepsilon_t^d}{\alpha_1\gamma_2} - \frac{\varepsilon_t^e}{\gamma_2}.$$

(A.5)

Appendix B. Equilibrium solutions under robust control

The dynamic system (39) is decomposable since the dynamics of expected inflation is independent of that of asset prices. We then use equation (34) to find the solution of π_t and $E_t \pi_{t+1}$. Using the method of indeterminate coefficients (McCallum, 1983), we assume that:

$$\pi_t = \varphi_0 + \varphi_1 \varepsilon_t^\pi, \tag{B.1}$$

and therefore,

$$E_t \pi_{t+1} = \varphi_0 + \varphi_1 E_t \varepsilon_{t+1}^\pi. \tag{B.2}$$

According to equation (4), it follows under the hypothesis of rational expectations:

$$E_t \varepsilon_{t+1}^\pi = \rho_s E_t \varepsilon_t^\pi + E_t e_{t+1}^\pi = \rho_s \varepsilon_t^\pi. \tag{B.3}$$

Using the result given in (B.3) in the assumed solution of the expected future inflation (B.2), we obtain:

$$E_t \pi_{t+1} = \varphi_0 + \varphi_1 \rho_s \varepsilon_t^\pi. \tag{B.4}$$

Substituting $E_t \pi_{t+1}$, defined by equation (B.4), into equation (34) yields:

$$\pi_t = \frac{\theta_h}{\theta_h - \lambda + \theta_h \lambda \delta^2} \left[\beta \varphi_0 + \frac{-\lambda + \theta_h \lambda \delta^2}{\theta_h} \pi^T + \delta y^* - (\beta \varphi_1 \rho_s - 1)\varepsilon_t^\pi \right]. \tag{B.5}$$

Comparing equation (B.5) with $\pi_t = \varphi_0 + \varphi_1 \varepsilon_t^\pi$, we obtain the coefficients φ_0 and φ_1 as follows:

$$\varphi_0 = \frac{-\lambda + \theta_h \lambda \delta^2}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} \pi^T + \frac{\theta_h \delta}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} y^*, \tag{B.6}$$

$$\varphi_1 = -\frac{\theta_h}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s}. \tag{B.7}$$

Then the solutions of π_t and $E_t \pi_{t+1}$ are given by:

$$\pi_t = \frac{-\lambda + \theta_h \lambda \delta^2}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} \pi^T + \frac{\theta_h \delta}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} y^* + \frac{\theta_h}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s} \varepsilon_t^\pi, \quad \text{(B.8)}$$

$$E_t \pi_{t+1} = \frac{-\lambda + \theta_h \lambda \delta^2}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} \pi^T + \frac{\theta_h \delta}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} y^* + \frac{\theta_h \rho_s}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s} \varepsilon_t^\pi. \quad \text{(B.9)}$$

It is straightforward to solve for y_t and $E_t y_{t+1}$ in substituting the solutions of π_t and $E_t \pi_{t+1}$ given by (B.8) and (B.9) respectively into equation (29) and the conditional expectations of equation (29) for period t+1:

$$y_t = \frac{(1-\beta)\lambda \delta \theta_h}{(\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta)} \pi^T + \frac{\theta_h(1-\beta) - \lambda}{(\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta)} y^* - \frac{\lambda \delta \theta_h}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s} \varepsilon_t^\pi. \quad \text{(B.10)}$$

$$E_t y_{t+1} = \frac{(1-\beta)\lambda \delta \theta_h}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} \pi^T + \frac{\theta_h(1-\beta) - \lambda}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} y^* - \frac{\lambda \delta \theta_h \rho_s}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s} \varepsilon_t^\pi. \quad \text{(B.11)}$$

In order to find the equilibrium solution of current and expected future asset prices, A_t and $E_t A_{t+1}$ respectively, we use the reduced form of asset pricing equation. The latter is obtained in using equations (27), (29), (37) and (A.1) to eliminate y_t, $E_t y_{t+1}$ and w_t in equation (15)[9]. Then, substituting the solutions of π_t and $E_t \pi_{t+1}$, given by equations (B.8) and (B.9), into the resulting asset pricing equation yields:

---

[9] We can alternatively use equation (A.5), but then we have to do some algebra to rewrite it in the form of equation (B.12).

$$A_t = \frac{\alpha_1 \gamma_2}{\alpha_1 + \alpha_2} E_t A_{t+1}$$

$$+ \gamma_1 y^* - \gamma_1 \lambda \delta \left[\frac{-\lambda + \theta_h \lambda \delta^2}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} \pi^T + \frac{\theta_h \delta}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_1 \beta} y^* + \frac{\theta_h}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s} \varepsilon_t^\pi \right] + \gamma_1 \lambda \delta \pi^T - \gamma_1 \varepsilon_t^\pi$$

$$- \frac{1}{\alpha_1} \left(\frac{\lambda \delta \theta_h \beta}{\theta_h - \lambda + \theta_h \lambda \delta^2} - \lambda \delta \right) \left[\frac{-\lambda + \theta_h \lambda \delta^2}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} \pi^T + \frac{\theta_h \delta}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta} y^* + \frac{\theta_h \rho_s}{\theta_h - \lambda + \theta_h \lambda \delta^2 - \theta_h \beta \rho_s} \varepsilon_t^\pi \right]$$

$$- \frac{1}{\alpha_1} \left[\frac{\lambda \delta \theta_h}{\theta_h - \lambda + \theta_h \lambda \delta^2} (\delta y^* + \varepsilon_t^\pi - \pi^T) + \lambda \delta \pi^T + \varepsilon_t^d \right] + \varepsilon_t^e$$

(B.12)

Rearranging the terms in equation (B.12) and after some simplifications, we obtain the difference equation of asset prices as follows:

$$A_t = \frac{\alpha_1 \gamma_2}{\alpha_1 + \alpha_2} E_t A_{t+1} + \frac{\gamma_1 \lambda \delta \theta_h (1 - \beta)}{\theta_h (1 - \beta) - \lambda + \theta_h \lambda \delta^2} \pi^T + \frac{\gamma_1 [\theta_h (1 - \beta) - \lambda]}{\theta_h (1 - \beta) - \lambda + \theta_h \lambda \delta^2} y^*$$

$$- \frac{1}{\alpha_1} \varepsilon_t^d + \varepsilon_t^e - \frac{\lambda \delta \theta_h + \gamma_1 \alpha_1 (\theta_h - \lambda + \theta_h \lambda \delta + \theta_h \lambda \delta^2) - (\gamma_1 \alpha_1 \beta + \lambda \delta) \theta_h \rho_s}{\alpha_1 [\theta_h (1 - \beta \rho_s) - \lambda + \theta_h \lambda \delta^2]} \varepsilon_t^\pi.$$

(B.13)

Under the hypothesis of rational expectations, we assume that the solutions of current and expected future asset prices take the following form:

$$A_t = \chi_0 + \chi_1 \varepsilon_t^d + \chi_2 \varepsilon_t^e + \chi_3 \varepsilon_t^\pi,$$

(B.14)

$$A_{t+1} = \chi_0 + \chi_1 \varepsilon_{t+1}^d + \chi_2 \varepsilon_{t+1}^e + \chi_3 \varepsilon_{t+1}^\pi.$$

(B.15)

According to equation (4), we can write

$$E_t \varepsilon_{t+1}^\pi = \rho_s E_t \varepsilon_t^\pi + E_t e_{t+1}^\pi = \rho_s \varepsilon_t^\pi,$$

(B.16)

$$E_t \varepsilon_{t+1}^d = \rho_d E_t \varepsilon_t^d + E_t e_{t+1}^d = \rho_d \varepsilon_t^d;$$

(B.17)

$$E_t \varepsilon_{t+1}^e = \rho_e E_t \varepsilon_t^e + E_t e_{t+1}^e = \rho_e \varepsilon_t^e.$$

(B.18)

Taking conditional expectations of equation (B.15) and using then equations (B.16)-(B.18) to eliminate $E_t \varepsilon_{t+1}^\pi$ $E_t \varepsilon_{t+1}^d$ and $E_t \varepsilon_{t+1}^e$ in the resulting equation give:

$$E_t A_{t+1} = \chi_0 + \chi_1 \rho_d \varepsilon_t^d + \chi_2 \rho_e \varepsilon_t^e + \chi_3 \rho_s \varepsilon_t^\pi. \tag{B.19}$$

Substituting $E_t A_{t+1}$ defined by (B.19) into equation (B.13) leads to

$$A_t = \frac{\alpha_1 \gamma_2}{\alpha_1 + \alpha_2}(\chi_0 + \chi_1 \rho_d \varepsilon_t^d + \chi_2 \rho_e \varepsilon_t^e + \chi_3 \rho_s \varepsilon_t^\pi) + \frac{\gamma_1 \lambda \delta \theta_h (1-\beta)}{\theta_h (1-\beta) - \lambda + \theta_h \lambda \delta^2} \pi^T + \frac{\gamma_1 [\theta_h (1-\beta) - \lambda]}{\theta_h (1-\beta) - \lambda + \theta_h \lambda \delta^2} y^*$$
$$- \frac{1}{\alpha_1} \varepsilon_t^d + \varepsilon_t^e - \frac{\lambda \delta \theta_h + \gamma_1 \alpha_1 (\theta_h - \lambda + \theta_h \lambda \delta + \theta_h \lambda \delta^2) - (\gamma_1 \alpha_1 \beta + \lambda \delta) \theta_h \rho_s}{\alpha_1 [\theta_h (1-\beta \rho_s) - \lambda + \theta_h \lambda \delta^2]} \varepsilon_t^\pi. \tag{B.20}$$

Rearranging the terms in equation (B.20), we have:

$$A_t = \frac{\alpha_1 \gamma_2}{\alpha_1 + \alpha_2} \chi_0 + \frac{\gamma_1 \lambda \delta \theta_h (1-\beta)}{\theta_h (1-\beta) - \lambda + \theta_h \lambda \delta^2} \pi^T + \frac{\gamma_1 [\theta_h (1-\beta) - \lambda]}{\theta_h (1-\beta) - \lambda + \theta_h \lambda \delta^2} y^* + (\frac{\alpha_1 \gamma_2}{\alpha_1 + \alpha_2} \chi_1 \rho_d - \frac{1}{\alpha_1}) \varepsilon_t^d$$
$$+ (\frac{\alpha_1 \gamma_2}{\alpha_1 + \alpha_2} \chi_2 \rho_e + 1) \varepsilon_t^e - \left\{ \frac{\lambda \delta \theta_h + \gamma_1 \alpha_1 (\theta_h - \lambda + \theta_h \lambda \delta + \theta_h \lambda \delta^2) - (\gamma_1 \alpha_1 \beta + \lambda \delta) \theta_h \rho_s}{\alpha_1 [\theta_h (1-\beta \rho_s) - \lambda + \theta_h \lambda \delta^2]} - \frac{\alpha_1 \gamma_2}{\alpha_1 + \alpha_2} \chi_3 \rho_s \right\} \varepsilon_t^\pi. \tag{B.21}$$

Comparing equation (B.21) with equation (B.14) leads to:

$$\chi_0 = \frac{\alpha_1 \gamma_2}{\alpha_1 + \alpha_2} \chi_0 + \frac{\gamma_1 \lambda \delta \theta_h (1-\beta)}{\theta_h (1-\beta) - \lambda + \theta_h \lambda \delta^2} \pi^T + \frac{\gamma_1 [\theta_h (1-\beta) - \lambda]}{\theta_h (1-\beta) - \lambda + \theta_h \lambda \delta^2} y^*, \tag{B.22}$$

$$\chi_1 = (\frac{\alpha_1 \gamma_2}{\alpha_1 + \alpha_2} \chi_1 \rho_d - \frac{1}{\alpha_1}), \tag{B.23}$$

$$\chi_2 = (\frac{\alpha_1 \gamma_2}{\alpha_1 + \alpha_2} \chi_2 \rho_e + 1), \tag{B.24}$$

$$\chi_3 = -\frac{\lambda \delta \theta_h + \gamma_1 \alpha_1 (\theta_h - \lambda + \theta_h \lambda \delta + \theta_h \lambda \delta^2) - (\gamma_1 \alpha_1 \beta + \lambda \delta) \theta_h \rho_s}{\alpha_1 [\theta_h (1-\beta \rho_s) - \lambda + \theta_h \lambda \delta^2]} + \frac{\alpha_1 \gamma_2}{\alpha_1 + \alpha_2} \chi_3 \rho_s. \tag{B.25}$$

Solving equations (B.22)-(B.25) yields the solution of the indeterminate coefficients as follows:

$$\chi_0 = \frac{(\alpha_1 + \alpha_2)\{\gamma_1 \lambda \delta \theta_h (1-\beta) \pi^T + \gamma_1 [\theta_h (1-\beta) - \lambda] y^*\}}{(\alpha_1 + \alpha_2 - \alpha_1 \gamma_2)[\theta_h (1-\beta) - \lambda + \theta_h \lambda \delta^2]}, \tag{B.26}$$

$$\chi_1 = -\frac{\alpha_1 + \alpha_2}{\alpha_1(\alpha_1 + \alpha_2 - \alpha_1\gamma_2\rho_d)}, \tag{B.27}$$

$$\chi_2 = \frac{\alpha_1 + \alpha_2}{\alpha_1 + \alpha_2 - \alpha_1\gamma_2\rho_e}, \tag{B.28}$$

$$\chi_3 = -\frac{(\alpha_1 + \alpha_2)[\lambda\delta\theta_h + \gamma_1\alpha_1(\theta_h - \lambda + \theta_h\lambda\delta + \theta_h\lambda\delta^2) - (\gamma_1\alpha_1\beta + \lambda\delta)\theta_h\rho_s]}{\alpha_1(\alpha_1 + \alpha_2 - \alpha_1\gamma_2\rho_s)[\theta_h(1 - \beta\rho_s) - \lambda + \theta_h\lambda\delta^2]}. \tag{B.29}$$

Then, substituting the solutions of χ_0, χ_1, χ_2 and χ_3 into equations (B.14) and (B.15) leads to the equilibrium solutions for A_t and $E_t A_{t+1}$:

$$A_t = \frac{(\alpha_1 + \alpha_2)\{\gamma_1\lambda\delta\theta_h(1 - \beta)\pi^T + \gamma_1[\theta_h(1 - \beta) - \lambda]y^*\}}{(\alpha_1 + \alpha_2 - \alpha_1\gamma_2)[\theta_h(1 - \beta) - \lambda + \theta_h\lambda\delta^2]} - \frac{\alpha_1 + \alpha_2}{\alpha_1(\alpha_1 + \alpha_2 - \alpha_1\gamma_2\rho_d)}\varepsilon_t^d$$
$$+ \frac{\alpha_1 + \alpha_2}{\alpha_1 + \alpha_2 - \alpha_1\gamma_2\rho_e}\varepsilon_t^e - \frac{(\alpha_1 + \alpha_2)[\lambda\delta\theta_h + \gamma_1\alpha_1(\theta_h - \lambda + \theta_h\lambda\delta + \theta_h\lambda\delta^2) - (\gamma_1\alpha_1\beta + \lambda\delta)\theta_h\rho_s]}{\alpha_1(\alpha_1 + \alpha_2 - \alpha_1\gamma_2\rho_s)[\theta_h(1 - \beta\rho_s) - \lambda + \theta_h\lambda\delta^2]}\varepsilon_t^\pi.$$

$$\tag{B.30}$$

$$E_t A_{t+1} = \frac{\gamma_1\lambda\delta\theta_1(\alpha_1 + \alpha_2)(1 - \beta)\pi^T + \gamma_1(\alpha_1 + \alpha_2)[\theta_1(1 - \beta) - \lambda]y^*}{(\alpha_1 + \alpha_2 - \alpha_1\gamma_2)(\theta_1 - \lambda + \theta_1\lambda\delta^2 - \theta_1\beta)} - \frac{(\alpha_1 + \alpha_2)\rho_d}{\alpha_1(\alpha_1 + \alpha_2 - \alpha_1\gamma_2\rho_d)}\varepsilon_t^d$$
$$+ \frac{(\alpha_1 + \alpha_2)\rho_e}{\alpha_1 + \alpha_2 - \alpha_1\gamma_2\rho_e}\varepsilon_t^e - \frac{(\alpha_1 + \alpha_2)[\lambda\delta\theta_h + \gamma_1\alpha_1(\theta_h - \lambda + \theta_h\lambda\delta + \theta_h\lambda\delta^2) - (\gamma_1\alpha_1\beta + \lambda\delta)\theta_h\rho_s]}{\alpha_1(\alpha_1 + \alpha_2 - \alpha_1\gamma_2\rho_s)[\theta_h(1 - \beta\rho_s) - \lambda + \theta_h\lambda\delta^2]}\rho_s\varepsilon_t^\pi.$$

$$\tag{B.31}$$

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