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"Modelling the transaction role of money and the essentiality of money in a hyperinflation context"

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Abstract

This paper uses an analytical approach and the precise definition of money essentiality given by Scheinkman (1980) with the aim to establish a formal theoretical link between the possibility of hyperinflationary paths and the concept of money essentiality. In this respect the paper contributes to the understanding of the well known failure of Cagan based inflationary finance models to produce explosive hyperinflation. We consider two standard optimizing monetary models representing alternative ways of modelling the transaction role of money. The paper considers a money-in-the-utility-function model and a cash-in-advance model where representative agent's preferences are represented by general utility functions. We show that modelling monetary hyperinflation with perfect foresight is closely linked to the concept of money essentiality as defined by Scheinkman (1980). The possibility of explosive monetary hyperinflation in a perfect foresight inflationary finance model always relies on a sufficient level of money essentiality. The main contribution of this paper is to show that, whether in a cash-in-advance or in a money-in-the-utility-function framework, this sufficient level of money essentiality does not depend on the specific way, cash-in-advance or moneyin-the-utility-function, of modelling the role of money as a medium of exchange.

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1. Introduction

Classical studies of hyperinflationary episodes provided an account of the stylized facts of hyperinflations on the basis of several observed European experiences (Bresciani-Turroni, 1937; Cagan, 1956; Sargent, 1982). These main stylized facts characterize hyperinflation as a speeding up inflation unstable dynamic process where real money balances tend to vanish and the public deficit is financed by issuing money: these processes can be qualified as monetary hyperinflations². Extreme inflation dramatically change economic exchange patterns compared to low-inflation inflation periods. The rapid depreciation of money during hyperinflation induces agents to spend money as soon as they have got it (Casella and Feinstein, 1990). Hyperinflation induces instability of relative price movements leading to large uncertainty about the outcomes of long-term contracts (Tang and Wang, 1993). As a consequence, hyperinflation decreases credit transactions and in general the use of long term contracts. As pointed out by Gutierrez and Vazquez (2004), this implies that money becomes more essential for purchasing goods during hyperinflation than during stable periods.

The concept of money essentiality has been precisely defined by Scheinkman (1980). According to Scheinkman (1980), the definition of money essentiality relates to the evolution of inflation tax collected by government when the rate of inflation explodes. Money is considered as essential if the inflation tax collected by the government does not tend to zero when the rate of inflation explodes. Therefore, the hyperinflation process is closely related to money essentiality and the inflation tax. Consistently with its salient stylized facts traditional models of hyperinflation view hyperinflation as the result of an inflationary finance policy. These inflationary finance models, such as Evans and Yarrow (1981) or Bruno and Fischer (1990), relying on the famous Cagan (1956) money demand, consider hyperinflation as a speeding up inflation process driven by an accelerating rise in the money supply as a means of raising revenues for the government by using the inflation tax. However, since the 'surprising monetarist arithmetic' analysed in Buiter (1987) it is known that under perfect foresight these models are fundamentally flawed because they are not capable of generating accelerating inflation³.

Given the well known failure of Cagan based inflationary finance models to produce explosive hyperinflation, Gutierrez and Vazquez (2004) investigate the characterization of agent's preferences compatible with explosive hyperinflation. They show that hyperinflationary dynamics derived from standard optimizing monetary models are consistent with a characterization of hyperinflation as an explosive process. Possible hyperinflationary paths arise naturally in a particular basic cash-in-advance model (henceforth called CIA model) where money is assumed strictly essential for transactions. However, in a specific money-in-the-utility-function model (henceforth called MIUF model) they show that possible hyperinflationary paths are more likely when the transaction role of money becomes more important. They conclude that a CIA model is 'a sensible approach to study hyperinflation since huge inflation rates cause credit transactions to vanish and money to become strictly essential for purchasing goods'.

This paper uses an analytical approach and the precise definition of money essentiality given by Scheinkman (1980) with the aim to establish a formal theoretical link between the

 $^{^2}$ This paper is not about speculative hyperinflations which are the focus of other works such as Brock (1975), Obstfeld and Rogoff (1983), Barbosa and da Cunha (2003) for instance. Speculative hyperinflations, as defined by Obstfeld and Rogoff (1983), are explosive price-level paths unrelated to monetary growth.

³ Evans (1995) and Vazquez (1998) provide a survey about the literature concerning this failure.

possibility of hyperinflationary paths and the concept of money essentiality. In this respect the paper contributes to the understanding of the well known failure of Cagan based inflationary finance models to produce explosive hyperinflation. We extend to more generality the two standard optimizing monetary models considered in Gutierrez and Vazquez (2004) which represent alternative ways of modelling the transaction role of money. The paper considers a MIUF model and a CIA model where representative agent's preferences are represented by general utility functions. We show that modelling monetary hyperinflation with perfect foresight is closely linked to the concept of money essentiality as defined by Scheinkman (1980). The possibility of explosive monetary hyperinflation in a perfect foresight inflationary finance model always relies on a sufficient level of money essentiality. The main contribution of this paper is to show that, whether in a CIA or in a MIUF framework, this sufficient level of money essentiality is always conveyed by the representative agent's preferences represented by its utility function and does not depend on the specific way, CIA or MIUF, of modelling the role of money as a medium of exchange.

The paper is organized in the following way. In section 2, we consider an inflationary finance optimizing MIUF model where representative agent's preferences are represented by a general utility function to relate the possibility of generating explosive monetary hyperinflations and the concept of money essentiality in the sense of Scheinkman (1980). In section 3, we consider an inflationary finance optimizing CIA model where representative agent's preferences are represented by a general utility function to establish, again, the dependence of the possibility of generating explosive monetary hyperinflations on the concept of money essentiality. Interestingly, section 3 shows that money essentiality is not conveyed by the CIA specific way of modelling the transaction role of money but that it is conveyed by the agent's preferences. Section 4 summarizes the results.

2. MIUF economy, hyperinflation and money essentiality

The optimizing monetary models considered in this paper assume a continuous time model where the economy consists of a large number of identical infinitely-lived forward looking households endowed with perfect foresight. Population is constant and its size is normalized to unity for convenience. There is no uncertainty. Each household has a non-produced constant endowment y > 0 of the non-storable consumption good per unit of time.

In the MIUF model the role of money as a medium of exchange is assumed to be captured by introducing real money balances into the household utility function. The set up draws on Sidrauski (1967) and Brock (1975).

The representative household utility at time 0 is

$$\int_{0}^{\infty} U(c_t, m_t) e^{-rt} dt .$$
(1)

The instantaneous utility function is continuous, twice differentiable on \mathbb{R}^2_+ , increasing and strictly concave in c_t , the household's consumption at time t, and $m_t = \frac{M_t}{P_t}$ his holdings of real monetary balances (*M* is the nominal stock of money, *P* is the price level). The rate r is

the subjective discount rate, which, following Calvo (1987), is assumed to be equal to the real rate of interest. Financial wealth and the nominal interest rate are defined as

$$\omega_t = m_t + b_t,$$

$$i_t = r + \pi_t,$$

respectively, where b_t denotes real per capita government debt, π_t is the inflation rate. The household's budget constraint is

$$\dot{\boldsymbol{\omega}} = \boldsymbol{y}_t - \boldsymbol{\tau}_t + \boldsymbol{r}\boldsymbol{\omega}_t - \left(\boldsymbol{c}_t + \boldsymbol{i}_t\boldsymbol{m}_t\right),\tag{2}$$

where τ_t is a lump-sum tax assumed to be constant. The household's optimization problem leads to the following first-order condition:

$$r + \pi_{t} = \frac{U'_{m}(c_{t}, m_{t})}{U'_{c}(c_{t}, m_{t})},$$
(3)

where $U'_{c}(c_{t},m_{t})$ is constant with respect to time because the instantaneous rate of time preference is equal to the real rate of interest. Condition (3) requires that at each moment the nominal rate of interest be equal to the marginal rate of substitution of consumption for money. It implicitly defines a demand for money as a function of the nominal interest rate *i*. Assuming $U''_{cm}(c,m) \ge 0$, the strict concavity of the utility function *U* ensures that *m* and *i* are related in a negative fashion. The optimum solution is completed by the transversality condition:

$$\lim_{t\to\infty} \left[e^{-rt} U_c'(c_t, m_t) \omega_t \right] = 0.$$
⁽⁴⁾

The equilibrium condition in the goods market is

$$y = c_t + g , (5)$$

where g is the constant per capita government expenditure In usual inflationary finance models a constant per capita share of government's budget deficit, d, is financed by issuing high-powered money:

$$d = \frac{M_t}{P_t} = \dot{m}_t + \pi_t m_t.$$
(6)

Substituting the value of π extracted from first-order equation (3) in the latter expression leads to the inflationary finance model dynamics described by the following law of motion for real cash balances:

$$\dot{m}_{t} = d - \left(\frac{U'_{m}(c_{t}, m_{t})}{U'_{c}(c_{t}, m_{t})} - r\right) m_{t}.$$
(7)

The differential equation (7) provides a complete characterization of real per-capita money balances dynamics which will be studied by using the technique of phase diagram on $[0;+\infty]$.

The main interesting point here is to examine whether this law of motion for real cash balances is able to produce monetary hyperinflation paths. A monetary hyperinflation path will be observed if the law of motion presents a path leading to a zero level of real cash balances. Therefore, the conditions for this kind of paths should be identified. As the mathematical function representing the law of motion is continuous (which is true with standard assumptions on U) this kind of paths will be observed as long as (dropping index time for convenience):

$$\lim_{m \to 0_+} \dot{m} < 0.$$
(8)

The calculation of $\lim_{m \to +\infty} \dot{m}$ will assess the existence of any steady state. Nevertheless, whatever the number of steady states, we are only interested in the paths starting at the left of the first possible steady state when the condition $\lim_{m \to +\infty} \dot{m} < 0$ is met.

At this stage a second highly important point should be made clear. According to Obstfeld and Rogoff (1983) in the context of speculative hyperinflations issue, any path leading to a zero value of real cash balances and crossing eventually the vertical axis at some finite point should be ruled out on grounds that such paths would not be feasible because the real stock of money would eventually become negative. However, we would rather follow the point made by Barbosa and Cunha (2003) who contested the Obstfeld and Rogoff (1983) approach by arguing that on such hyperinflationary paths when the real quantity of money reaches zero hyperinflation would have wiped out the value of money, the opportunity cost of holding money would have become infinite, and the economy would no longer be a monetary economy. Therefore, we follow the point made by Barbosa and Cunha (2003) and consider the monetary hyperinflation paths corresponding to the condition $\lim \dot{m} < 0$ as perfect

foresight competitive equilibrium paths.

Moreover, it's important to stress that the possible hyperinflationary paths are monetary hyperinflations because along these paths the rate of growth of the money supply explodes. Rewriting government budget constraint as

$$\frac{\dot{M}}{M} = \frac{d}{m},$$

we see that along the paths of continuously declining m, given that d > 0, the growth rate of money supply increases continuously.

In this respect, according to the law of motion (7), the possibility of explosive monetary hyperinflation will depend on the condition

$$\lim_{m \to 0_{+}} \left[\frac{U'_{m}(c,m)}{U'_{c}(c,m)} m \right] > d .$$
⁽⁹⁾

The latter condition is basically a condition about a sufficient level of money essentiality. Scheinkman (1980) defines the essentiality of money as the fact that "money is very necessary to the system". According to his definition, money essentiality relates to the evolution of inflation tax collected by government when the rate of inflation explodes. Money is considered as essential if the inflation tax collected by the government does not tend to zero when the rate of inflation explodes. From (6) we see that seigniorage obtained by printing money can be decomposed into two components, the change in the real stock of money and the inflation tax πm which can be written, according to equation (3):

$$\pi m = \left(\frac{U'_{m}(c,m)}{U'_{c}(c,m)} - r\right) m = \frac{U'_{m}(c,m)}{U'_{c}(c,m)} m - rm.$$
(10)

Then, when the rate of inflation explodes we consider

$$\lim_{m \to 0_{+}} \pi m = \lim_{m \to 0_{+}} \left[\frac{U'_{m}(c,m)}{U'_{c}(c,m)} m \right].$$
 (11)

Therefore, when $\lim_{m\to 0_+} \left[\frac{U'_m(c,m)}{U'_c(c,m)} m \right] > 0$ then $\lim_{m\to 0_+} \pi m > 0$ and money is essential. These findings enable us to formulate a first proposition.

Proposition 1: In a general MIUF optimizing monetary framework, explosive monetary hyperinflations are possible only if money is sufficiently essential that is $if \lim_{m \to 0_+} \left[\frac{U'_m(c,m)}{U'_n(c,m)} m \right] > d.$

Proof: The proof relies on the previous arguments and can be illustrated by the phase diagram depicted on Figure 1. The precise shape of the phase diagram depends on the first and second derivative of \dot{m} with respect to m. Other shapes than that depicted on Figure 1 could be possible for the phase locus. However, as the important point for the analysis conducted here insists on the condition for $\lim_{m\to 0_+} \dot{m} < 0$, our analysis focuses only on the paths leading to a zero value of real cash balances. If $\lim_{m\to +\infty} \dot{m} > 0$, the locus \dot{m} will cross the horizontal axis at least once. We consider here a unique unstable steady state m^* but the qualitative analysis for

hyperinflationary paths doesn't change in the case of more steady state m but the quantative analysis for hyperinflationary paths doesn't change in the case of more steady states. All paths originating at the right of m^* are hyperdeflationary paths that can be ruled out because violating the transversality condition (4). All paths starting to the left of m^* are monetary hyperinflations paths.



Figure 1: Monetary dynamics in a MIUF economy with money sufficiently essential

In the choice of a particular constant-relative-risk-aversion utility function like

$$U(c_t, m_t) = \frac{\left(c_t^{1-\omega} m_t^{\omega}\right)^{1-\alpha} - 1}{1-\alpha},$$
(12)

as in Gutierrez and Vazquez (2004), the parameter ω ($0 \le \omega \le 1$) is supposed measuring the transaction requirement of money. One can easily show that the latter utility function represents agent's preferences which comply with Proposition 1 requirement only if money is

sufficiently essential to the transactions that is if $\omega > \frac{d}{c+d}$.

Furthermore, considering the case where the utility function is additively separable in consumption and real cash balances:

$$U(c_t, m_t) = u(c_t) + v(m_t), \qquad (13)$$

where the functions u and v are increasing in their arguments and strictly concave, the condition (9) of Proposition 1 resumes to

$$\lim_{m \to 0_{+}} [mv'(m)] > du'(c) \,. \tag{14}$$

In the latter condition the value of u'(c) is constant with respect to time. Scheinkman (1980) identified the condition $\lim_{m\to 0_+} mv'(m) > 0$ to the essentiality of money. The condition (14), as a particular case of Proposition 1, states that the possibility of explosive monetary hyperinflation depends on a sufficient level of money essentiality which is conveyed by the utility function for money services.

3. Cash-In-Advance economy, hyperinflation and money essentiality

The framework of the CIA model considered in this section is the same as that considered in the previous section. The CIA model considered here differs from that of the MIUF in two aspects. First, the representative household's preferences are represented by the following utility at time 0:

$$\int_{0}^{\infty} e^{-rt} U(c_t) dt \,. \tag{15}$$

The function U is increasing and strictly concave in its argument, real good consumption. Second, in a CIA economy the role of money as a medium of exchange is captured by a cashin-advance constraint assuming that money holding is strictly essential to buy the consumption good. In order to consume c units of the consumption good at time t, the household must hold a stock of real cash balances, m, greater or equal to c:

$$m_t \ge c_t \tag{16}$$

Assuming the existence of an interior solution for c, and that the nominal interest rate i is greater than zero, meaning that money is return-dominated by government bond, it follows that CIA constraint (16) must hold with equality:

$$m_t = c_t \,. \tag{16}$$

The representative household optimization problem consisting of maximizing (15) subject to the constraints given by (2) and (16)' leads to the following first order condition

$$U'(m_t) = \lambda (1+i_t), \qquad (17)$$

where λ is the associated Lagrange multiplier which is constant with respect to time because the agent's rate of time preference equals the real rate of interest, and real cash balances will indirectly enter the utility function according to (16)'. Equation (17) characterizes a demand for real money balances decreasing with respect to the rate of inflation (or the cost of holding cash balances) because the utility function U is strictly concave. The transversality condition implies that

$$\lim_{t \to \infty} e^{-rt} \lambda \omega_t = 0.$$
 (18)

By using the definition of the nominal interest rate, the first order condition (17) can be written as follows:

$$\pi_t = \frac{U'(m_t) - \lambda (1+r)}{\lambda}.$$
(19)

As in usual inflationary finance models a constant per capita share of government's budget deficit, d, is financed by issuing high-powered money, the law of motion for real money balances in this CIA inflationary finance model will be given by combining (6) and (19), leading to

$$\dot{m}_{t} = d - \frac{1}{\lambda} \left(U'(m_{t}) - \lambda \left(1 + r \right) \right) m_{t}.$$
⁽²⁰⁾

On the basis of the methodology and the argumentation developed in section 2, the possibility of explosive monetary hyperinflation paths depends on condition (8) leading to the following condition in the CIA current economy (dropping the time index for convenience)

$$\lim_{m \to 0_+} \left[mU'(m) \right] > \lambda d . \tag{21}$$

In the same way as in section 2 in the framework of a MIUF economy, the condition (21) relates the possibility of explosive monetary hyperinflation to a sufficient level of money essentiality which is conveyed by the agent's preferences. The Scheinkman (1980) definition of money essentiality considers the evolution of inflation tax collected by government when the rate of inflation explodes. Money is considered as essential if the inflation tax collected by the government does not tend to zero when the rate of inflation explodes. According to (19), inflation tax is given by

$$\pi m = \left(\frac{U'(m) - \lambda(1+r)}{\lambda}\right) m.$$
(22)

Then, when the rate of inflation explodes we consider

$$\lim_{m \to 0_+} \left[\pi m \right] = \frac{1}{\lambda} \lim_{m \to 0_+} \left[m U'(m) \right].$$
⁽²³⁾

From the mathematical point of view it appears that the condition (21) allowing the model to generate possible monetary hyperinflations paths is exactly of the same kind as the condition (9) in the general MIUF model. The condition (21) is particularly similar to the condition (14) in the MIUF case with an additive separable utility function.

Proposition 2: In a CIA optimizing monetary framework with a general utility function, explosive monetary hyperinflations are possible only if money is sufficiently essential that is if $\lim_{m \to 0} [mU'(m)] > \lambda d.$

Proof: The proof relies on previous arguments.

The possibility of monetary hyperinflation paths is again a discussion about a sufficient level of money essentiality, and is not linked to the specificity of the CIA framework. The CIA framework is not sufficient in itself to ensure the possibility of explosive hyperinflations paths.

Considering the choice of a particular constant-relative-risk-aversion utility function like

$$U(c) = \frac{c^{1-\alpha} - 1}{1-\alpha},$$
(24)

with $\alpha > 1$, as in Gutierrez and Vazquez (2004), implies agent's preferences which are compatible with the possibility of explosive monetary hyperinflation. The latter utility function complies with the condition (20) of Proposition 2 since we have

$$\lim_{m \to 0_{+}} \left[mU'(m) \right] = +\infty > \lambda d \tag{25}$$

The interesting results obtained in Gutierrez and Vazquez (2004) don't rely on the specificity of the CIA framework but rather on a sufficient level of money essentiality which is conveyed by the choice of the utility function given by (24). CIA constraint doesn't convey by itself sufficient money essentiality.

4. Conclusion

Several studies of economic exchange during hyperinflationary episodes report that the use of money is more essential during extreme inflations than during stable periods. Extreme inflation dramatically decreases credit transactions and in general the use of long term contracts. This paper studies the theoretical relation between the possibility of explosive monetary hyperinflation and the concept of money essentiality as defined by Scheinkman (1980). Hyperinflationary dynamics are characterized using two standard optimizing monetary models representing alternative ways of modelling the transaction role of money: a money-in-the-utility-function model and a cash-in-advance model with general representative agents' preferences.

The first contribution of the paper is to show that in both models the possibility of explosive monetary hyperinflations, defined as speeding up inflation processes driven by an inflationary finance policy, depends on a sufficient level of money essentiality. Money has to be sufficiently essential to the system, in the precise sense given by Scheinkman (1980), in order to make possible the occurrence of explosive monetary hyperinflation in inflationary finance models. The sufficient money essentiality requirement is consistent with the fact that money becomes more essential for purchasing goods during hyperinflation. It is also consistent with the design of inflationary finance models of hyperinflation since the government needs the money to be essential to the system in order to get sufficient inflation tax when inflation explodes.

The second contribution of the paper is to show that the essentiality of money requirement doesn't depend on the specific way used to model the role of money as a medium of exchange, cash-in-advance or money-in-the-utility-function. We show that the interesting results of Gutierrez and Vazquez (2004) can be thought as particular cases of our analysis, but we depart from the latter by showing that the cash-in-advance framework in itself does not convey the essentiality of money. The sufficient level of money essentiality is, in both models, showed to be conveyed by the representative agents' preferences. In this respect, the paper may contribute to the understanding of the well known Cagan inflationary finance model failure with perfect foresight and may stimulate further research for the choice of an appropriate demand for real cash balances in hyperinflation contexts.

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