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## « Best-reply matching and the centipede game »

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# Best-reply matching and the centipede game

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## Abstract

In their paper on Best-Reply Matching (BRM), Droste, Kosfeld & Voorneveld (2003) obtained quite intuitive results for the centipede game. In this short paper we first show that these results derive from the application of their criterion to the reduced normal form of the game. Then we prove that applying their criterion to the normal form of the game leads to different results. Third we propose an extension of Droste, Kosfeld & Voorneveld's criterion, which leads to the same results in both the reduced normal form and the normal form of a game. This extension leads to a larger set of behaviors, including the Subgame Perfect Nash equilibrium but also a limited rationality behavior that strongly sustains the continuation of the game.

## Résumé

Droste, Kosfeld et Voorneveld (2003), en appliquant le concept de Best-Reply Matching (BRM) au jeu du mille-pattes, ont obtenu des résultats nouveaux, à savoir que la probabilité de poursuivre le jeu chute avec le nombre d'étapes restantes et qu'elle tend vers 1 au début du jeu lorsque le nombre d'étapes devient grand. Dans ce working paper, nous montrons dans un premier temps que leurs résultats découlent d'une application du concept de BRM à la forme normale réduite du jeu du mille-pattes. Nous montrons ensuite qu'une application du concept de BRM à la forme normale du jeu mène à des résultats sensiblement différents : la probabilité de poursuivre le jeu reste une fonction décroissante

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du nombre d'étapes restantes, mais elle tend vers  $\frac{1}{2}$  seulement au début du jeu quand le nombre d'étapes devient grand. Enfin nous montrons qu'une modification de la définition du BRM équilibre permet non seulement d'obtenir les mêmes résultats en forme normale et en forme normale réduite, mais elle permet d'étendre l'ensemble des comportements d'équilibre. Cet ensemble inclut comme comportements extrêmes l'équilibre de Nash parfait en sous-jeu et un comportement simple de rationalité limitée, qui conduit chaque acteur à poursuivre le jeu avec une probabilité inversement proportionnelle au nombre d'étapes restantes plus 1.

**JEL classification:** C72

**Keywords:** Best-Reply Matching, centipede game, reduced normal form, normal form, Subgame Perfect Nash equilibrium.

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## 1. Introduction

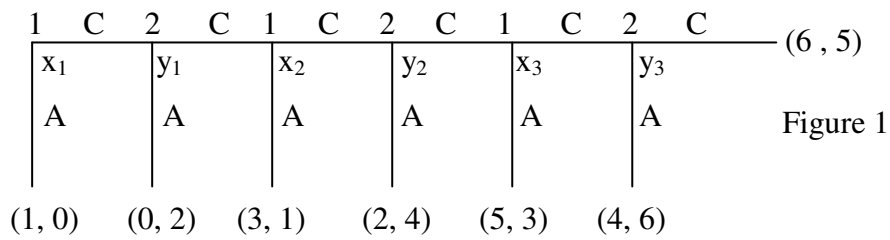
Droste, Kosfeld and Voorneveld (DKV, 2003) applied the concept of best-reply matching to the centipede game. They defined the concept of best-reply matching for games in normal form, yet they used a different, more local method, to get their quite intuitive results, which are: the probability to pursue the centipede game is a decreasing function of the remaining steps of the game, and the probability to pursue the game at its beginning goes to  $1$  when the number of steps becomes large. In this paper, we first show that their results are the ones obtained if one applies their normal form BRM definition to the *reduced normal form* of the game. Second we show that if one applies their definition to the *normal form* of the game, then the results perceptibly differ. To put it more precisely, the probability to pursue the centipede game is still a decreasing function of the remaining steps of the game, but the probability to pursue the game at its beginning goes to  $1/2$  when the number of steps becomes large. Third we show that an extension of DKV's definition of BRM allows to reconcile both results. This extension ensures that both the reduced

normal form and the normal form lead to the same BRM issues. It enlarges the set of BRM equilibria in the centipede game. At one extreme, the set contains the Subgame Perfect Nash equilibrium which leads both players to stop the game at the first decision node. At the other extreme, it contains a limited rationality behavior that leads each player, at each decision node, to pursue the game with probability 1 divided by the number of remaining decision nodes plus 1.

## 2. The reduced normal form and Droste, Kosfeld and Voorneveld's result

The centipede game has two main properties. The first is that the more the players continue (C) the game, the more the payoffs they get grow. Second, it is always better to stop (A) the game before the opponent stops the game. The ordinal concept of BRM exploits these two properties.

DKV work on the game given in figure 1.



*Legend of figure 1: the first, respectively the second, coordinate of each vector of payoffs is the payoff of player 1, respectively player 2.*

Let us recall DKV's BRM equilibrium concept:

### **BRM equilibrium (Kosfeld & al. 2002, Droste & al. 2003)**

Let  $G=(N, (S_i)_{i \in N}, (\succ_i)_{i \in N})$  be a game. A mixed strategy  $p$  is a (normal form) BRM equilibrium if for every player  $i \in N$  and for every pure strategy  $s_i \in S_i$ ,

$$p_i(s_i) = \sum_{s_{-i} \in B_i^{-1}(s_i)} \frac{1}{\text{Card } B_i(s_{-i})} p_i(s_{-i})$$

In a BRM equilibrium, the probability assigned to a pure strategy is linked to the number of times the opponents play the strategies to which this pure strategy is a best reply. So, if player  $i$ 's opponents play  $s_{-i}$  with probability  $p_{-i}(s_{-i})$ , and if the set of player  $i$ 's pure strategies that are best responses to  $s_{-i}$  is the subset  $B_i(s_{-i})$ , then each strategy of this subset is played with the probability  $p_{-i}(s_{-i})$  divided by the cardinal of  $B_i(s_{-i})$ . This concept carries on the concept of rationalizability developed by Bernheim (1984) and Pearce (1984), a pure strategy  $s_i$  being rationalizable if there exists a pure strategy profile  $s_{-i}$  played by the opponents to which  $s_i$  is a best response. DKV go further: they observe that, if the opponents often play  $s_{-i}$ , then  $s_i$  often becomes the best response, and therefore they argue that it is rational for player  $i$  to often play  $s_i$ . Consequently they require that, if  $s_{-i}$  is played with probability  $p_{-i}$ ,  $s_i$  should be played with the same probability (if  $s_i$  is the only best reply to  $s_{-i}$ ). Given that the same condition is checked for each pure strategy, each player's probability distribution (on pure strategies) is justified by the opponents' probability distributions, which ensures a strong behavior consistency.

The reduced normal form of the centipede game is given in matrix 1, together with the best-reply table (table 1).

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squares left to the second diagonal are filled with  $B_2$ . In the game with  $n$  decision nodes for each player, there are  $n + 1$  strategies for each player. We define them as follows:

For each player,  $\underline{c}$  consists to always pursue the game;  $s_i$ ,  $i$  from 1 to  $n$ , consists to pursue the game at each decision node before decision node  $i$  and to stop the game at decision node  $i$ .  $p_i$ , respectively  $q_i$ , is the probability that player 1, respectively player 2, assigns to  $s_i$ ,  $i$  from 1 to  $n$ .  $p_{n+1}$ , respectively  $q_{n+1}$ , is the probability that player 1, respectively player 2, assigns to  $\underline{c}$ .

It immediately follows:

$\forall i$  from 1 to  $n+1$ ,  $p_i = q_i$ , because player 1 is right stopping the game at node  $i$  if and only if player 2 stops the game at node  $i$ .

One also observes that:

$q_{n+1} = p_1/(n+1)$  because always pursuing the game is one among player 2's  $n+1$  best replies to player 1's strategy that consists to stop the game at her first decision node.

$q_n = p_1/(n+1) + p_{n+1} = q_{n+1} + q_{n+1} = 2q_{n+1}$  because stopping the game at node  $n$  is one among player 2's  $n+1$  best replies to player 1's strategy that consists to stop the game at her first decision node, and it is the only best response to player 1's strategy  $\underline{c}$ .

In the same way, one obtains:

$$q_{n-1} = p_1/(n+1) + p_n = q_{n+1} + q_n = 3q_{n+1}$$

$$q_{n-2} = p_1/(n+1) + p_{n-1} = q_{n+1} + q_{n-1} = 4q_{n+1}$$

and so on, with  $q_i = (n-i+2)q_{n+1}$ , for  $i$  from  $n-3$  to 1.

Given that the  $q_i$ ,  $i$  from 1 to  $n+1$ , sum to 1, it derives:

$$(1+2+3+\dots+(n+1)) q_{n+1} = 1 \quad \text{i.e.} \quad q_{n+1} = 2/[(n+1)(n+2)] = p_{n+1}$$

It follows:  $\forall i$  from 1 to  $n+1$ ,  $q_i = p_i = (n-i+2)2/[(n+1)(n+2)]$

Player 1's Kuhn equivalent behavioral strategies are given by:

$$\begin{aligned} \pi_{x_n}(A) &= \text{probability that player 1 plays A given that she is at node } x_n \\ &= p_n/(p_n+p_{n+1}) = 4/(4+2) = 2/3 \end{aligned}$$

In the same way, one gets:

$$\forall k \text{ from 1 to } n, \quad \pi_{x_k}(A) = \text{probability that player 1 plays A given that she is at node } x_k$$

$$= p_k / \left( \sum_{j=k}^{n+1} p_j \right) = 2 / (n-k+3)$$

This is exactly the result DKV obtain with a more local approach.

The same results holds for  $\pi_{y_k}(A)$ , the probability that player 2 stops the game at node  $y_k$ , for  $k$  from 1 to  $n$ .

#### DKV's result (2003)

In the reduced normal form centipede game with  $n$  decision nodes for each player, each player, at his decision node  $k$ , stops the game with probability  $2/(n-k+3)$ . One immediately observes that this probability grows in  $k$ , and that the probability to stop the game at the first node, equal to  $2/(n+2)$ , *goes to 0 when  $n$  becomes large*.

### 3. Best-reply matching in the normal form game

Table 2 is the best-reply table for the normal form game associated to the centipede game of figure 1.

|                 |       | 2         |           |           |           |       |       |       |       |
|-----------------|-------|-----------|-----------|-----------|-----------|-------|-------|-------|-------|
|                 |       | $A_1$     | $A_1$     | $A_1$     | $A_1$     | $C_1$ | $C_1$ | $C_1$ | $C_1$ |
|                 |       | $A_2$     | $A_2$     | $C_2$     | $C_2$     | $A_2$ | $A_2$ | $C_2$ | $C_2$ |
|                 |       | $A_3$     | $C_3$     | $A_3$     | $C_3$     | $A_3$ | $C_3$ | $A_3$ | $C_3$ |
|                 |       | $q_1$     | $q_2$     | $q_3$     | $q_4$     | $q_5$ | $q_6$ | $q_7$ | $q_8$ |
| $A_1 A_2 A_3$   | $p_1$ | $B_1 B_2$ | $B_1 B_2$ | $B_1 B_2$ | $B_1 B_2$ | $B_2$ | $B_2$ | $B_2$ | $B_2$ |
| $A_1 A_2 C_3$   | $p_2$ | $B_1 B_2$ | $B_1 B_2$ | $B_1 B_2$ | $B_1 B_2$ | $B_2$ | $B_2$ | $B_2$ | $B_2$ |
| $A_1 C_2 A_3$   | $p_3$ | $B_1 B_2$ | $B_1 B_2$ | $B_1 B_2$ | $B_1 B_2$ | $B_2$ | $B_2$ | $B_2$ | $B_2$ |
| 1 $A_1 C_2 C_3$ | $p_4$ | $B_1 B_2$ | $B_1 B_2$ | $B_1 B_2$ | $B_1 B_2$ | $B_2$ | $B_2$ | $B_2$ | $B_2$ |
| $C_1 A_2 A_3$   | $p_5$ | $B_2$     | $B_2$     | $B_2$     | $B_2$     | $B_1$ | $B_1$ |       |       |
| $C_1 A_2 C_3$   | $p_6$ | $B_2$     | $B_2$     | $B_2$     | $B_2$     | $B_1$ | $B_1$ |       |       |
| $C_1 C_2 A_3$   | $p_7$ |           |           |           |           | $B_2$ | $B_2$ | $B_1$ |       |
| $C_1 C_2 C_3$   | $p_8$ |           |           |           |           |       |       | $B_2$ | $B_1$ |

Table 2

The structure of table 2 easily generalizes to the centipede game with  $n$  decision nodes for each player. In the  $n$  decision nodes centipede game, each player has  $2^n$  strategies that we class in  $(n+1)$  different families:

- The family  $S_1$  of strategies  $s_i$ , with  $i$  from 1 to  $2^{n-1}$ , is the set of pure strategies such that the player stops the game at the first decision node.
  - The family  $S_2$  of strategies  $s_i$ , with  $i$  from  $2^{n-1}+1$  to  $2^{n-1}+2^{n-2}$  is the set of pure strategies in which the player pursues the game at decision node 1 but stops it at decision node 2.
  - More generally the family  $S_k$  of strategies  $s_i$ , with  $i$  from  $2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^{n-k+1}+1$  to  $2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^{n-k+1}+2^{n-k}$  is the set of pure strategies such that the player pursues the game at each decision node before  $k$  and stops it at decision node  $k$ , with  $n>k>1$ .
  - The singleton family  $S_n$  which contains the strategy  $s_{2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^1+1}$  which consists to pursue the game at each decision node except for the last.
  - The singleton family  $\underline{C}$  which contains the strategy  $s_{2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^1+2}$ , i.e.  $s_{2^n}$ , which leads the player to pursue the game at each decision node.
- Player 1, respectively player 2, assigns probability  $p_i$ , respectively  $q_i$ , to the strategy  $s_i$ , with  $i$  from 1 to  $2^n$ .

We obtain the following result:

**Proposition 1**

In the *normal form* centipede game with  $n$  decision nodes for each player,  $p_{2^n}=q_{2^n}=1/(2^{n+1}-1)$  and  $q_i=p_i=2/(2^{n+1}-1)$  for  $i$  from 1 to  $2^n-1$ . It follows that each player stops the game at decision node  $k$  with probability  $2^{n-k+1}/(2^{n-k+2}-1)$ , for  $k$  from 1 to  $n$ . This probability is increasing in  $k$ , but the probability to stop the game at the first decision node, i.e.  $2^n/(2^{n+1}-1)$ , *is always higher than  $1/2$  and goes to  $1/2$  when  $n$  becomes large.*

**Proof :** see appendix 1

It follows that the application of DKV's criterion to the centipede game in normal form leads to assign higher values to strategies that lead to an earlier end of the game than the application of DKV's criterion to the reduced normal form of the game. Indeed it is easy to check that  $2/(n-i+3) < 2^{n-i+1}/(2^{n-i+2}-1)$  for any  $i$  from 1 to  $n-1$ , and that  $2/(n-i+3) = 2^{n-i+1}/(2^{n-i+2}-1) = 2/3$  for  $i=n$ . It derives that *the reduced normal form under evaluates the probability to stop the game at early steps (in comparison with the normal form).*

Let us illustrate the results for  $n = 3$ .

Applying DKV's criterion to the normal form leads to

$$\pi_{x_1}(A) = \pi_{y_1}(A) = 8/15 > 1/2, \pi_{x_2}(A) = \pi_{y_2}(A) = 4/7, \pi_{x_3}(A) = \pi_{y_3}(A) = 2/3.$$

Applying DKV's criterion to the reduced normal form of the game leads to:

$$\pi_{x_1}(A) = \pi_{y_1}(A) = 2/5 < 1/2, \pi_{x_2}(A) = \pi_{y_2}(A) = 1/2, \pi_{x_3}(A) = \pi_{y_3}(A) = 2/3.$$

These results are illustrated in figures 2b and 2c (see section 5).

#### 4. A slight modification in BRM's definition: a way to reconcile the normal form and the reduced normal form

In DKV's criterion, a player is supposed to share equally a probability between all the strategies that are best responses. For example, suppose that  $A_1$  and  $B_1$  are best replies for player 1 only if player 2 plays  $A_2$ . Suppose also that player 2 plays  $A_2$  with probability  $q$ . If so, the BRM concept assigns probability  $q/2$  to  $A_1$  and to  $B_1$ . Yet there is no reason to divide equally  $q$  between  $A_1$  and  $B_1$ . A more neutral way to cope with indifference consists in examining the whole set of equilibrium possibilities, in which the probability assigned to  $A_1$  is  $p$ , with  $0 \leq p \leq q$ , the probability assigned to  $B_1$  being  $q-p$ , i.e. to adopt the more general definition given in Umbhauer (2007):

##### New normal form BRM equilibrium (Umbhauer 2007)

Let  $G = (N, (S_i)_{i \in N}, (\succ_i)_{i \in N})$  be a game. A mixed strategy  $p$  is a new normal form BRM equilibrium if for every player  $i \in N$  and for every pure strategy  $s_i \in S_i$ :

$$p_i(s_i) = \sum_{s_{-i} \in B_i^{-1}(s_i)} \delta_{s_i} p_{-i}(s_{-i})$$

with  $\delta_{s_i} \in [0, 1]$  for any  $s_i$  belonging to  $B_i(s_{-i})$  and  $\sum_{s_i \in B_i(s_{-i})} \delta_{s_i} = 1$

With this more general definition, one reconciles the results obtained for the normal form with the results obtained for the reduced normal form.

**Proposition 2**

The new BRM criterion selects the same issues whether one works with the reduced normal form or with the normal form of a game.

**Proof: see appendix 2**

For example, for  $n=3$ , when we work on the normal form of the game, instead of using the set of equations:

$$p_i = q_i \quad i \text{ from } 1 \text{ to } 8,$$

$$q_1 = q_2 = q_3 = q_4 = (p_1 + p_2 + p_3 + p_4)/8 + (p_5 + p_6)/4$$

$$q_5 = q_6 = (p_1 + p_2 + p_3 + p_4)/8 + p_7/2$$

$$q_7 = (p_1 + p_2 + p_3 + p_4)/8 + p_8$$

$$q_8 = (p_1 + p_2 + p_3 + p_4)/8$$

we can use the set of equations :

$$q_1 = q_2 = q_3 = q_4 = (p_1 + p_2 + p_3 + p_4)/16 + (p_5 + p_6)/4$$

$$q_5 = q_6 = (p_1 + p_2 + p_3 + p_4)/8 + p_7/2$$

$$q_7 = (p_1 + p_2 + p_3 + p_4)/4 + p_8$$

$$q_8 = (p_1 + p_2 + p_3 + p_4)/4$$

This new way to (unequally) share the probability  $(p_1 + p_2 + p_3 + p_4)$  between player 2's best replies to player 1's strategy that consists to stop the game at the first decision node, ensures that the Kuhn equivalent behavioral strategies will be the same than the ones obtained for the reduced normal form (when the probabilities, in the reduced normal form, are equally shared in case of indifference).

## **5. A larger range of BRM equilibria including the Subgame Perfect Nash equilibrium and a simple limited rationality behavior**

Of course, this more neutral way to cope with indifference enlarges the number of obtained BRM equilibria. It is interesting to look at the extreme behaviors profiles induced by this modification.

**Proposition 3**

At one extreme, one of the new BRM equilibria leads to the same issue than the Subgame Perfect Nash equilibrium.

**Proof:**

This proposition immediately follows from the fact that each pure strategy Nash equilibrium is a new normal form BRM equilibrium (see Umbhauer 2007). ☺

Yet, it is interesting to prove the result directly by looking for the weights assigned to each strategy in case of indifference (i.e. the way to share a probability in case of indifference) that lead to the result.

In the centipede game in reduced normal form, indifference only happens in the following situation: player 2 is indifferent between all his strategies when player 1 stops the game at her first decision node. To get the Subgame Perfect Nash equilibrium behavior, it is sufficient that player 2, if player 1 stops the game at her first decision node, puts weight 1 on the strategy that also consists to stop the game at his first decision node.

Hence, in the reduced normal form, it is sufficient to only assign  $p_1$  to  $q_1$  instead of sharing  $p_1$  equally between all player 2's strategies: this means that, albeit all player 2's strategies are best responses when player 1 stops the game at  $x_1$ , only player 2's strategy that consists to stop the game at  $y_1$  will be played in this event. Hence we get the new system of equations:

$$p_i = q_i \quad i \text{ from } 1 \text{ to } n+1$$

$$q_{n+1} = 0, p_1 = 0$$

$$q_n = 0, p_1 + p_{n+1} = q_{n+1} = 0$$

$$q_{n-1} = 0, p_1 + p_n = q_n = 0$$

and so on, till to  $i=2$

$$q_2 = 0, p_1 + p_3 = q_3 = 0$$

$$\text{and } q_1 = 1, p_1 + p_2 = p_1 + q_2 = p_1.$$

It immediately follows that  $q_1=p_1=1$  and  $p_i=q_i=0$  for  $i$  from 2 to  $n+1$ . This behavior leads to the same issue than the Subgame Perfect Nash equilibrium.

In the normal form, we share equally  $\sum_{i=1}^{2^{n-1}} p_i$  only among the  $q_i$ , with  $i$  from 1 to  $2^{n-1}$ , i. e. we

also suppose that, albeit all player 2's strategies are best responses when player 1 stops the game at  $x_1$ , only player 2's strategies that consists to stop the game at  $y_1$  will be played in this event. So we get the new set of equations<sup>2</sup>:

$$p_i=q_i \text{ for } i \text{ from } 1 \text{ to } 2^n$$

$$q_{2^n}=0$$

$$q_{2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^1+1}=p_{2^n}=q_{2^n}=0$$

For  $i$  from  $2^{n-1}+\dots+2^2+1$  to  $2^{n-1}+\dots+2^2+2$

$$q_i = \frac{p_{2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^1+1}}{2} = \frac{q_{2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^1+1}}{2} = 0$$

And so on, till to  $i$  from  $2^{n-1}+1$  to  $2^{n-1}+2^{n-2}$

$$q_i = \frac{p_{2^{n-1}+2^{n-2}+1}}{2} = \frac{q_{2^{n-1}+2^{n-2}+1}}{2} = 0$$

By contrast, for  $i$  from 1 to  $2^{n-1}$ , we get:

$$q_i = \frac{\sum_{i=1}^{2^{n-1}} p_i}{2^{n-1}} + p_{2^{n-1}+1} / 2 = \frac{\sum_{i=1}^{2^{n-1}} p_i}{2^{n-1}} + q_{2^{n-1}+1} / 2 = \frac{\sum_{i=1}^{2^{n-1}} p_i}{2^{n-1}} = p_i.$$

Hence  $q_i=p_i=1/2^{n-1}$  for  $i$  from 1 to  $2^{n-1}$  and  $q_i=p_i=0$  for  $i$  from  $2^{n-1}+1$  to  $2^n$ , which means that both player 1 and player 2 stop the game at their first decision node, like in the Subgame Perfect Nash equilibrium.

#### Proposition 4

At the other extreme, the set of new BRM equilibria contains a quite simple behavior defined as follows: in the centipede game with  $n$  decision nodes for each player, each

<sup>2</sup> We still assign the same probability to each strategy of a same family.

player stops the game at decision node  $k$ ,  $k$  from 1 to  $n$ , with probability  $1/(n-k+2)$ . In other words, each player stops the game with probability 1 divided by the number of remaining decision nodes plus one. It namely follows that each player stops the game at the first decision node with probability  $1/(n+1)$ , and stops the game at the last decision node with probability  $1/2$ . It is immediate that the probability to stop the game at the first decision node goes to 0 when  $n$  becomes large.

**Proof:** see appendix 3

This behavior derives from a system of weights that leads player 2 to never stop the game in case of indifference, i.e. when player 1 stops the game at her first decision node.

This result is interesting for two reasons.

First, *like the Subgame Perfect Nash equilibrium, it is very simple*: a player who knows that there remain  $x$  decision nodes (including the one at which he is playing), simply stops the game with probability  $1/(x+1)$ .

Second this result is *compatible with an easy (limited rationality) reasoning*, which is different for player 1 and 2, and which goes as follows:

Player 1, when she is at her last decision node, is happy to pursue the game if player 2, at his last decision node, pursues the game, and she is happy to stop the game in the other case. Hence she is happy to stop the game only in *one among two configurations*, which simply leads her to stop the game with probability  $1/2$ .

More generally, when player 1 is at her decision node  $k$ , she is happy to stop the game if player 2 stops the game at his decision node  $k$ . In all the other configurations, i.e. if player 2 stops the game at node  $k+1$ , stops at node  $k+2$ , ... stops at node  $n$ , or never stops the game, i.e. in  $(n-k+1)$  configurations, she is happy to pursue the game. So she is happy to stop the game in only *one among  $(n-k+2)$  configurations*, which simply leads her to stop the game with probability  $1/(n-k+2)$ .

As regards player 2, a simple explanation of his behavior consists in saying that, given that he plays after player 1, *he simply mimics player 1*. This is again not a silly behavior (even if it leads to an apparently inconsistent behavior at his last decision node) for at least two reasons. First, player 2 is in a similar situation than player 1 (except for the last decision node), but he observes player 1's behavior at decision node  $k$  before playing at his own

decision node k; this makes mimetism possible. Second, his mimetism expresses a kind of ***reward/reprisals behavior***, which is all the more interesting that it leads both player to high payoffs.

Let us illustrate, for  $n=3$ , four possible behaviors:

|          |                |          |                |          |                |          |                |          |                |          |                |        |
|----------|----------------|----------|----------------|----------|----------------|----------|----------------|----------|----------------|----------|----------------|--------|
| 1        | <u>0</u> C     | 2        | <u>0</u> C     | 1        | <u>1-u</u> C   | 2        | <u>1-u</u> C   | 1        | <u>1-u</u> C   | 2        | <u>1-u</u> C   | (6, 5) |
|          | x <sub>1</sub> |          | y <sub>1</sub> |          | x <sub>2</sub> |          | y <sub>2</sub> |          | x <sub>3</sub> |          | y <sub>3</sub> |        |
|          | A              |          | A              |          | A              |          | A              |          | A              |          | A              |        |
| <u>1</u> |                | <u>1</u> |                | <u>u</u> |                | <u>u</u> |                | <u>u</u> |                | <u>u</u> |                |        |
| (1, 0)   |                | (0, 2)   |                | (3, 1)   |                | (2, 4)   |                | (5, 3)   |                | (4, 6)   |                |        |

Figure 2a  
Extreme stopping  
behavior

|             |                |             |                |            |                |            |                |            |                |            |                |        |
|-------------|----------------|-------------|----------------|------------|----------------|------------|----------------|------------|----------------|------------|----------------|--------|
| 1           | <u>7/15</u> C  | 2           | <u>7/15</u> C  | 1          | <u>3/7</u> C   | 2          | <u>3/7</u> C   | 1          | <u>1/3</u> C   | 2          | <u>1/3</u> C   | (6, 5) |
|             | x <sub>1</sub> |             | y <sub>1</sub> |            | x <sub>2</sub> |            | y <sub>2</sub> |            | x <sub>3</sub> |            | y <sub>3</sub> |        |
|             | A              |             | A              |            | A              |            | A              |            | A              |            | A              |        |
| <u>8/15</u> |                | <u>8/15</u> |                | <u>4/7</u> |                | <u>4/7</u> |                | <u>2/3</u> |                | <u>2/3</u> |                |        |
| (1, 0)      |                | (0, 2)      |                | (3, 1)     |                | (2, 4)     |                | (5, 3)     |                | (4, 6)     |                |        |

Figure 2b

|            |                |            |                |            |                |            |                |            |                |            |                |        |
|------------|----------------|------------|----------------|------------|----------------|------------|----------------|------------|----------------|------------|----------------|--------|
| 1          | <u>3/5</u> C   | 2          | <u>3/5</u> C   | 1          | <u>1/2</u> C   | 2          | <u>1/2</u> C   | 1          | <u>1/3</u> C   | 2          | <u>1/3</u> C   | (6, 5) |
|            | x <sub>1</sub> |            | y <sub>1</sub> |            | x <sub>2</sub> |            | y <sub>2</sub> |            | x <sub>3</sub> |            | y <sub>3</sub> |        |
| <u>2/5</u> | A              | <u>2/5</u> | A              | <u>1/2</u> | A              | <u>1/2</u> | A              | <u>2/3</u> | A              | <u>2/3</u> | A              |        |
| (1, 0)     |                | (0, 2)     |                | (3, 1)     |                | (2, 4)     |                | (5, 3)     |                | (4, 6)     |                |        |

Figure 2c

|            |                |            |                |            |                |            |                |            |                |            |                |        |
|------------|----------------|------------|----------------|------------|----------------|------------|----------------|------------|----------------|------------|----------------|--------|
| 1          | <u>3/4</u> C   | 2          | <u>3/4</u> C   | 1          | <u>2/3</u> C   | 2          | <u>2/3</u> C   | 1          | <u>1/2</u> C   | 2          | <u>1/2</u> C   | (6, 5) |
|            | x <sub>1</sub> |            | y <sub>1</sub> |            | x <sub>2</sub> |            | y <sub>2</sub> |            | x <sub>3</sub> |            | y <sub>3</sub> |        |
|            | A              |            | A              |            | A              |            | A              |            | A              |            | A              |        |
| <u>1/4</u> |                | <u>1/4</u> |                | <u>1/3</u> |                | <u>1/3</u> |                | <u>1/2</u> |                | <u>1/2</u> |                |        |
| (1, 0)     |                | (0, 2)     |                | (3, 1)     |                | (2, 4)     |                | (5, 3)     |                | (4, 6)     |                |        |

Figure 2d  
Extreme pursuing  
behavior

*Legend of figures 2: the underlined numbers are the probabilities of the associated actions. u means undetermined.*

## 6. Conclusion

It is interesting to observe that the alternative consistency behind best-reply matching leads to a set of interesting behaviors in the centipede game. It enlarges the number of consistent behaviors without eliminating the Subgame Perfect Nash equilibrium, which is one extreme (new) BRM equilibrium. It gives rise to a large range of probabilities of pursuing the game at the different decision nodes, each system of probabilities corresponding to one way of dealing with indifference. The way that most favours the continuation of the game leads to a very easy behavior that, in addition to have the consistency of a BRM equilibrium, respects a kind of limited rationality doubled by mimetic behavior. The fact that BRM, by contrast to subgame perfection, leads to behaviors closer to the behaviors expected in the centipede game, proves that a consistency criterion build on rationalizability can lead to meaningful behavior. The same conclusion has been observed in other well-know games like Akerlof's market for lemons (see Umbhauer 2007).

## References

- Bernheim, B.D., 1984. Rationalizable strategic behavior. *Econometrica* 52, 1007-1028.
- Droste, E., Kosfeld, M., Voorneveld, M., 2003. Best-reply matching in games. *Mathematical Social Sciences* 46, 291-309.
- Kosfeld, M., Droste, E., and Voorneveld, M., 2002. A myopic adjustment process leading to best-reply matching. *Games and Economic Behavior* 40, 270-298.
- Pearce, D.G., 1984. Rationalizable strategic behavior and the problem of perfection. *Econometrica* 52, 1029-1050.
- Droste, E., Kosfeld, M., Voorneveld, M., 2003. Best-reply matching in games. *Mathematical Social Sciences* 46, 291-309.
- Umbhauer, G., 2007. Best-reply matching in an experience good model. <http://ssrn.com/author=490799>.

## Appendix 1

Let us prove proposition 1.

First, we observe that the probability assigned to a strategy of a family  $S_i$ ,  $i$  from 1 to  $n-1$ , is the same than the probability assigned to another strategy of the same family. This is due to the fact that each strategy of a same family is justified in the same way. So, for example, each strategy of player 1 which leads her to continue before node  $k$  and to stop at node  $k$ , is justified by player 2's strategy that consists in pursuing the game before node  $k$  and stopping at node  $k$ .

Hence,  $p_i = p_j$  and  $q_i = q_j$  for  $i, j$  from 1 to  $2^{n-1}$ , and, more generally:

$$\forall k \text{ from } 2 \text{ to } n-1 \quad p_i = p_j \text{ and } q_i = q_j \text{ for any } i, j \text{ from } \sum_{j=2}^k 2^{n-j+1} + 1 \text{ to } \sum_{j=1}^k 2^{n-j}.$$

Second, we observe that:

$$\text{- For any } i \text{ from } 1 \text{ to } 2^{n-1}, \quad p_i = \frac{\sum_{i=1}^{2^{n-1}} q_i}{2^{n-1}} \quad \text{because stopping at the first decision node is player}$$

1's best reply if player 2 also stops the game at his first decision node. Hence  $p_i = q_i$ , given the first observation.

- More generally, for  $k$  from 2 to  $n-1$ , for any  $i$  from  $2^{n-1} + \dots + 2^{n-k+1} + 1$  to  $2^{n-1} + \dots + 2^{n-k+1}$

$$+ 2^{n-k}, \quad p_i = \frac{\sum_{i=2^{n-1} + \dots + 2^{n-k+1} + 1}^{2^{n-1} + \dots + 2^{n-k+1} + 2^{n-k}} q_i}{2^{n-k}} \quad \text{because continuing at each node before } k \text{ and stopping}$$

at node  $k$  is player 1's best reply if player 2 pursues the game at each node before  $k$  and stops it at node  $k$ . Hence,  $p_i = q_i$  given the first observation.

- Finally  $p_{2^{n-1} + 2^{n-2} + 2^{n-3} + \dots + 2^1 + 1} = q_{2^{n-1} + 2^{n-2} + 2^{n-3} + \dots + 2^1 + 1}$  because player 1 is best off only stopping the game at her last decision node if and only if player 2 only stops the game at his last decision node

And  $p_{2^n} = q_{2^n}$  because player 1 is right always pursuing the game if and only if player 2 always pursues the game.

It follows that  $p_i = q_i$  for any  $i$  from 1 to  $2^n$ .

In fact we exploit the square structure of the  $B_1$  (see the squares in table 2). Each player 1's strategy in a family  $S_k$  is justified by all the strategies of player 2's family  $S_k$ . Given that each strategy of a family is played with the same probability, and given that Card  $S_k$  is the same for both players, it automatically follows that the probability assigned by player 1 to a strategy in  $S_k$  is the same than the one assigned by player 2 to this strategy.

Third we turn to player 2. We have:

$$q_{2^n} = \frac{\sum_{i=1}^{2^{n-1}} p_i}{2^n} = p_1/2$$

$$q_{2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^1+1} = p_{2^n} + \frac{\sum_{i=1}^{2^{n-1}} p_i}{2^n} = q_{2^n} + p_1/2 = 2q_{2^n}$$

For  $i$  from  $2^{n-1}+\dots+2^2+1$  to  $2^{n-1}+\dots+2^2+2$

$$q_i = \frac{\sum_{i=1}^{2^{n-1}} p_i}{2^n} + \frac{p_{2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^1+1}}{2} = p_1/2 + \frac{q_{2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^1+1}}{2} = 2q_{2^n}$$

More generally, let us suppose that, for a  $k$  from 2 to  $n-2$ :

$$\forall i \text{ from } 2^{n-1}+\dots+2^{n-k}+1 \text{ to } 2^{n-1}+\dots+2^{n-k-1}, \quad q_i = p_i = 2q_{2^n}$$

and let us prove that :

$$\forall i \text{ from } 2^{n-1}+\dots+2^{n-k+1}+1 \text{ to } 2^{n-1}+\dots+2^{n-k}$$

$$q_i = p_i = 2q_{2^n}$$

We have:

$$\forall i \text{ from } 2^{n-1}+\dots+2^{n-k+1}+1 \text{ to } 2^{n-1}+\dots+2^{n-k}$$

$$\begin{aligned} q_i &= \frac{\sum_{i=1}^{2^{n-1}} p_i}{2^n} + \frac{\sum_{i=2^{n-1}+\dots+2^{n-k-1}+1}^{2^{n-1}+\dots+2^{n-k-1}} p_i}{2^{n-k}} = p_1/2 + p_{2^{n-1}+\dots+2^{n-k}+1}/2 \\ &= p_1/2 + q_{2^{n-1}+\dots+2^{n-k}+1}/2 \\ &= 2q_{2^n} \end{aligned}$$

Moreover:

$\forall i$  from 1 to  $2^{n-1}$

$$q_i = \frac{\sum_{i=1}^{2^{n-1}} p_i}{2^n} + \frac{\sum_{i=2^{n-1}+1}^{2^{n-1}+2^{n-2}} p_i}{2^{n-1}} = p_1/2 + p_{2^{n-1}+1}/2 = p_1/2 + q_{2^{n-1}+1}/2 = 2q_{2^n}$$

Hence:  $\forall i$  from 1 to  $2^n-1$ ,  $q_i = p_i = 2q_{2^n}$

From  $\sum_{i=1}^{2^n} q_i = 1$ , we deduce that:

$$(2^n-1)2q_{2^n} + q_{2^n} = 1, \text{ hence } q_{2^n} = 1/(2^{n+1}-1) = p_{2^n}$$

And  $q_i = p_i = 2/(2^{n+1}-1)$  for  $i$  from 1 to  $2^n-1$

The Kuhn equivalent behavioral strategies become:

$\forall k$  from 1 to  $n$

$$\pi_{x_k}(A) = \frac{p(\overbrace{CC \dots C}^{k-1}, A, \overbrace{** \dots *}^{n-k})}{p(\overbrace{CC \dots C}^{k-1}, \overbrace{*** \dots *}^{n-k+1})} \quad \text{where } p(x) \text{ is the probability of the event } x \text{ and a star}$$

means either action A or action C.

$$= \frac{2^{n-k} 2q_{2^n}}{(2^{n-k+1}-1)2q_{2^n} + q_{2^n}} = 2^{n-k+1}/(2^{n-k+2}-1).$$

It derives that  $\pi_{x_1}(A) = 2^n/(2^{n+1}-1)$  which is higher than  $1/2$  and goes to  $1/2$  when  $n$  goes to infinity. The same results hold for  $\pi_{y_k}(A)$  for  $k$  from 1 to  $n$ .

## Appendix 2

The proof of proposition 2 is given for 2 player games but it easily generalizes to  $n$  player games, given the linearity of the equations. So we only give the proof for 2 player games.

In this context, switching from the reduced normal form to the normal form amounts to adding identical lines and /or identical columns.

So let us start with a reduced normal form and let us add, without loss of generality, a line identical to line i. So we switch from matrix 1 to matrix 2. The three points symbolize the possible additional pure strategies.

|     |                  |                                      |                                      |     |                  |                |
|-----|------------------|--------------------------------------|--------------------------------------|-----|------------------|----------------|
|     |                  | 2                                    |                                      |     |                  |                |
|     |                  | D <sub>1</sub>                       | D <sub>2</sub>                       | ... |                  |                |
| 1   | A <sub>i</sub>   | (a <sub>i1</sub> , b <sub>i1</sub> ) | (a <sub>i2</sub> , b <sub>i2</sub> ) |     | p <sub>i</sub>   | B <sub>2</sub> |
| ... | ...              |                                      |                                      |     | ...              | B <sub>1</sub> |
|     |                  | matrix 1                             |                                      |     | table 1          |                |
|     |                  | 2                                    |                                      |     |                  |                |
|     |                  | D <sub>1</sub>                       | D <sub>2</sub>                       | ... |                  |                |
| 1   | A <sub>i</sub> ' | (a <sub>i1</sub> , b <sub>i1</sub> ) | (a <sub>i2</sub> , b <sub>i2</sub> ) |     | r <sub>i</sub> ' | B <sub>2</sub> |
| 1   | A <sub>i</sub>   | (a <sub>i1</sub> , b <sub>i1</sub> ) | (a <sub>i2</sub> , b <sub>i2</sub> ) |     | r <sub>i</sub>   | B <sub>2</sub> |
| ... | ...              |                                      |                                      |     | ...              |                |
|     |                  | matrix 2                             |                                      |     | table 2          |                |

Let us suppose, without loss of generality, that A<sub>i</sub>, and therefore A<sub>i</sub>', are best responses to D<sub>2</sub>, and that D<sub>1</sub> is a best response to A<sub>i</sub> and therefore to A<sub>i</sub>'. Adding a line identical to line i hence leads us to switch from the best-reply table 1 to the best-reply table 2.

Consequently, in the reduced normal form, we get the equation:

$$p_i = q_2 \alpha_{2i} + \dots$$

given that, on the one hand, there are possibly many best responses to D<sub>2</sub> so that  $\alpha_{2i}$  is the share of probability  $q_2$  assigned to A<sub>i</sub>, on the other hand A<sub>i</sub> may be a best reply to another strategy of player 2, which explains the three points.

We also get the equation:

$q_1 = p_i \alpha_{i1} + \dots$  given that, on the one hand, there are possibly many best responses to A<sub>i</sub> so that  $\alpha_{i1}$  is the share of probability  $p_i$  assigned to D<sub>1</sub>, on the other hand D<sub>1</sub> may be a best reply to another strategy of player 1, which explains the three points.

With the normal form, these equations become:

$$r_i = t_2 \beta_{2i} + \dots \quad \text{and} \quad r_i' = t_2 \beta_{2i'} + \dots$$

where  $\beta_{2i}$  and  $\beta_{2i'}$  are respectively the shares of probability  $t_2$  assigned to A<sub>i</sub> and A<sub>i</sub>' ,

and  $t_1 = r_i \beta_{i1} + r_i' \beta_{i'1} + \dots$  where  $\beta_{i1}$  and  $\beta_{i'1}$  are respectively the share of the probabilities  $r_i$  and  $r_i'$  assigned to D<sub>1</sub>.

The expected payoffs of player 1 and player 2 are in matrix 1  $a_{i1} p_i q_1 + a_{i2} p_i q_2 + \dots$  and  $b_{i1} p_i q_1 + b_{i2} p_i q_2 + \dots$ .

These payoffs become in matrix 2:

$$a_{i1} (r_i + r_{i'}) t_1 + a_{i2} (r_i + r_{i'}) t_2 + \dots \text{ and } b_{i1} (r_i + r_{i'}) t_1 + b_{i2} (r_i + r_{i'}) t_2 + \dots$$

Let us start with given weights  $\alpha$  in the reduced normal form. To get the same payoffs in matrix 2, one observes that, if  $t_2=q_2$ , it is sufficient to set  $\beta_{2i} + \beta_{2i'} = \alpha_{2i}$  (a similar constraint holding for other weights if  $A_i$  is a best response to other actions than  $D_2$ ), so that  $p_i=r_i+r_{i'}$ , and to set  $\beta_{i1}=\beta_{i'1}=\alpha_{i1}$ , so that  $t_1=q_1$ . Moreover, given that the definition of  $t_2$  and  $q_2$  do not depend on  $A_i$ , the equality between  $t_2$  and  $q_2$  automatically follows, provided one set  $r_j=p_j$  for any player 1's action  $A_j$ ,  $j$  different from  $i$  and  $i'$ .

Reciprocally, let us start with given weights  $\beta$  in the normal form. In that case, first, it is sufficient to set  $\alpha_{2i} = \beta_{2i} + \beta_{2i'}$ , in order to get  $r_i+r_{i'}=p_i$  (given  $t_2=q_2$ ). Second, in order to get  $q_1=t_1$ , we set  $p_i\alpha_{i1} = r_i\beta_{i1} + r_{i'}\beta_{i'1}$ . So we get:

$$\alpha_{i1} = (r_i\beta_{i1} + r_{i'}\beta_{i'1})/p_i = \beta_{i1} r_i / (r_i+r_{i'}) + \beta_{i'1} r_{i'} / (r_i+r_{i'}).$$

Hence  $\alpha_{i1}$  is a weighed mean of  $\beta_{i1}$  and  $\beta_{i'1}$ , which is possible ( $0 \leq \alpha_{i1} \leq 1$ ).

Hence for each BRM equilibrium in the reduced normal form there exists a BRM equilibrium in the normal form that leads to the same issue, and vice versa.

### Appendix 3

Let us first prove proposition 4 by working on the reduced normal form.

To this aim we simply assign  $p_1$  to  $q_{n+1}$  instead of sharing it equally between all player 2's strategies: in words, albeit all player 2's strategies are best responses in the event where player 1 stops the game at  $x_1$ , only player 2's strategy that consists to never stop the game will be played in this event. Hence we get the new system of equations:

$$p_i=q_i \text{ for } i \text{ from } 1 \text{ to } n+1$$

$$q_{n+1}=p_1 \mathbf{1} = p_1$$

$$q_n=p_1 \mathbf{0} + p_{n+1} = q_{n+1} = p_1$$

$$q_{n-1}=p_1 \mathbf{0} + p_n = q_n = p_1$$

and so on, till to  $q_1$ :

$$q_1=p_1 \mathbf{0} + p_2 = q_2 = p_1$$

Summing to 1 leads to  $(n+1) p_1=1$ , hence

$$p_i=q_i=1/(n+1) \text{ for } i \text{ from } 1 \text{ to } n+1.$$

The Kuhn equivalent behavioral strategies are given by:

$$\forall k \text{ from } 1 \text{ to } n, \quad \pi_{x_k}(A) = p_k / \left( \sum_{j=k}^{n+1} p_j \right) = 1 / (n-k+2)$$

Let us now prove proposition 4 by working on the normal form (even if this proof is not necessary (see proposition 2)).

To this aim, we assign  $\sum_{i=1}^{2^{n-1}} p_i$  to  $q_{2^n}$ , so we suppose that, albeit all player 2's strategies are

best responses in the event where player 1 stops the game at  $x_1$ , only player 2's strategy that consists to never stop the game will be played in this event. We now get the new set of equations:

$$p_i = q_i \text{ for } i \text{ from } 1 \text{ to } 2^n$$

$$q_{2^n} = \sum_{i=1}^{2^{n-1}} p_i = 2^{n-1} p_1$$

$$q_{2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^1+1} = \sum_{i=1}^{2^{n-1}} p_i + p_{2^n} = q_{2^n} = 2^{n-1} p_1$$

For  $i$  from  $2^{n-1}+\dots+2^2+1$  to  $2^{n-1}+\dots+2^2+2$

$$q_i = \sum_{i=1}^{2^{n-1}} p_i + \frac{p_{2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^1+1}}{2} = \frac{q_{2^{n-1}+2^{n-2}+2^{n-3}+\dots+2^1+1}}{2} = 2^{n-2} p_1$$

More generally, let us suppose that:

$$\forall i \text{ from } 2^{n-1}+\dots+2^{n-k}+1 \text{ to } 2^{n-1}+\dots+2^{n-k-1}, \quad q_i = p_i = 2^k p_1 \quad \text{for a } k \text{ between } 2 \text{ and } n-1,$$

and let us prove that :

$$\forall i \text{ from } 2^{n-1}+\dots+2^{n-k+1}+1 \text{ to } 2^{n-1}+\dots+2^{n-k}, \quad q_i = p_i = 2^{k-1} p_1$$

We have:

$$\forall i \text{ from } 2^{n-1}+\dots+2^{n-k+1}+1 \text{ to } 2^{n-1}+\dots+2^{n-k}$$

$$q_i = \sum_{i=1}^{2^{n-1}} p_i + \frac{\sum_{i=2^{n-1}+\dots+2^{n-k}+1}^{2^{n-1}+\dots+2^{n-k-1}} p_i}{2^{n-k}} = p_{2^{n-1}+\dots+2^{n-k}+1} / 2 = 2^k p_1 / 2 = 2^{k-1} p_1 .$$

For  $i$  from 1 to  $2^{n-1}$ , we get  $q_i = \frac{\sum_{i=1}^{2^{n-1}} p_i}{2^{n-1}} = \frac{2^{n-1} + 2^{n-2}}{2^{n-1}} = p_{2^{n-1}+1}/2 = 2p_1/2 = p_1$

It follows that, for  $k$  from 2 to  $n$

$$\sum_{i=2^{n-1}+...+2^{n-k+1}+1}^{2^{n-1}+...+2^{n-k}} q_i = 2^{n-k} q_{2^{n-1}+...+2^{n-k+1}+1}^1 = 2^{n-k} 2^{k-1} p_1 = 2^{n-1} p_1$$

$$\text{We also have } q_{2^n} = \frac{\sum_{i=1}^{2^{n-1}} p_i}{2^{n-1}} = 2^{n-1} p_1 \quad \text{and} \quad \sum_{i=1}^{2^{n-1}} q_i = 2^{n-1} p_1$$

Hence, summing all the  $q_i$ ,  $i$  from 1 to  $2^n$ , leads to:

$$(n+1)2^{n-1} p_1 = 1$$

It follows:

For  $k$  from 2 to  $n$ , for  $i$  from  $2^{n-1}+...+2^{n-k+1}+1$  to  $2^{n-1}+...+2^{n-k}$

$$q_i = p_i = 2^{k-1} p_1 = 2^{k-1} / [(n+1) 2^{n-1}] = 1 / [2^{n-k}(n+1)]$$

$$\text{Moreover, } q_{2^n} = p_{2^n} = 1/(n+1)$$

$$\text{and, for } i \text{ from 1 to } 2^{n-1}, q_i = p_i = 1/[2^{n-1}(n+1)]$$

It follows:

$\forall k$  from 1 to  $n$

$$\begin{aligned} \pi_{x_k}(A) &= \frac{\sum_{i=2^{n-1}+...+2^{n-k+1}+1}^{2^{n-1}+...+2^{n-k}} p_i}{\left( \sum_{j=k}^n \left( \sum_{i=2^{n-1}+...+2^{n-j+1}+1}^{2^{n-1}+...+2^{n-j}} p_i \right) + 2^{n-1} p_1 \right)} \\ &= (2^{n-1} p_1) / [(n-k+2) 2^{n-1} p_1] = 1/(n-k+2) \end{aligned}$$

The same result holds for  $\pi_{y_k}(A)$  for  $k$  from 1 to  $n$ .

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