Bureau d'économie théorique et appliquée (BETA)
UMR 7522

Documents de travail

« Best–reply matching in Akerlof's market for lemons »

Auteur

Gisèle UMBHAUER

Document de Travail n° 2007–**10**

Février 2007

Faculté des sciences économiques et de gestion
Pôle européen de gestion et d'économie (PEGE)
61 avenue de la Forêt Noire
F-67085 Strasbourg Cedex

Secétariat du BETA
Christine DEMANGE
Tél. : (33) 03 90 24 20 69
Fax : (33) 03 90 24 20 70
demange@cournot.u–strasbg.fr
http://cournot.u-strasbg.fr/beta

Best-reply matching in Akerlof's market for lemons

Gisèle Umbhauer[12]

Bureau d'Economie Théorique et Appliquée
Université Louis Pasteur, Strasbourg France

First version November 2006
Revised version February 2007

Abstract

The paper studies Akerlof's market for lemons in a new way. We firstly construct mixed Perfect Bayesian Nash equilibria in which all qualities are sold on the market even if the seller's strategy set is reduced to prices. Then we turn to the best-reply matching (BRM) approach developed by Droste, Kosfeld & Voorneveld (2003) for games in normal form. In a BRM equilibrium, the probability assigned by a player to a pure strategy is linked to the number of times this strategy is a best reply to the other players' played strategies. We extend this logic to signaling games in extensive form and apply the new obtained concept to Akerlof's model. This new concept leads to a very simple rule of behaviour, which is consistent, different from the Bayesian equilibrium behaviour, different from Akerlof's result, and can be socially efficient.

Résumé

L'article apporte un nouvel éclairage au marché des voitures d'occasion d'Akerlof. On montre dans un premier temps que le passage des stratégies pures aux stratégies mixtes suffit pour établir l'existence d'équilibres Bayesiens dans lesquels toutes les qualités sont vendues, même lorsque le vendeur ne dispose que du prix comme seule stratégie. On quitte ensuite la logique de Nash pour celle du Best-Reply Matching (BRM) introduite par Droste, Kosfeld et

[1] e-mail: umbhauer@cournot.u-strasbg.fr
[2] The author is grateful to Jacques Thépot for comments.

Voorneveld (2003) pour les jeux sous forme normale. Dans un BRM équilibre, la probabilité assignée à chaque stratégie est liée à la fréquence à laquelle elle est la meilleure réponse aux stratégies des autres acteurs. On propose une extension de ce critère d'équilibre aux jeux de signaux sous forme extensive tout en étudiant les difficultés conceptuelles soulevées par cette extension. Puis on applique le nouveau concept obtenu au modèle d'Akerlof. Cette application débouche sur une règle de comportements cohérente, généralisable et d'une très grande simplicité, qui assure la vente de toutes les qualités de bien et qui diffère profondément de celles obtenues avec le concept d'équilibre Bayesien.

JEL classification: C72 D82 L15
Keywords: best-reply matching, experience goods, signalling game, mixed Perfect Bayesian Equilibrium, extensive form, normal form

---

1. Introduction

In Akerlof's market for lemons, bad quality goods may throw high quality goods out of the market. Yet Akerlof's result rests on a pure strategy approach. So we show, in section 2, that switching to mixed Perfect Bayesian Equilibria (PBE) allows different quality experience goods to all be profitably sold on the market at different prices. In section 3, we comment the nature of these equilibria and compare them to pure strategy PBE in games with an additional strategic variable. In sections 4, 5, 6 and 7 we give up the mixed Nash equilibrium logic and turn to the Best-Reply Matching (BRM) equilibrium concept developed by Droste, Kosfeld & Voorneveld (2003) for games in normal form. In a BRM equilibrium, the probability that a player assigns to a pure strategy is linked to the number of times this strategy is a best reply to the strategies played by the other players. This concept displays a different consistency than the mixed Nash equilibrium one. In a BRM equilibrium, the probability distribution of player i is not the one that justifies the strategies played by the other players (as it is the case in the mixed Nash equilibrium), but it expresses the number of times each of her played strategy is a best reply. We argue that this consistency is realistic, given that players are generally more concerned with the justification of their own behavior, rather than with the justification of the other players' behavior. In section 4 and 5 we look for BRM equilibria in normal form games, before and after elimination of weakly dominated strategies: the results do not significantly differ in the studied games. In section 6, we extend the BRM equilibrium logic to signaling games in extensive form and apply the new obtained concept to Akerlof's model. We show that the switch from the normal form to the extensive form is not trivial from a logical point of view. The reason lies in the decentralization of the actions (and hence also in the decentralization of the justification of these actions) which is possible in the extensive form and precluded in the normal form. It follows that we do not get the same BRM equilibria in our experience good model, depending on whether we work with the extensive or the normal form of the game. In section 7 we focus on the simplified Akerlof's model with n prices $p_1 \ldots p_n$, such that the seller whose good is of quality t_i can only get a positive payoff by selling the good at prices p_j with j higher or equal than i . In this model the BRM logic for games in extensive form selects a very easy profile of strategies: each quality t_i is sold at each price p_j, with j higher or equal than i, with a same probability, and the consumer accepts each price with the probability 1 divided by the number of qualities possibly sold at this price. This behaviour is not only consistent with the BRM logic, but it is also very easy to learn and therefore to adopt. Finally, in section 8, we conclude on the social surplus allowed by the BRM logic in the price experience good model.

2. Different prices for different qualities

Akerlof's model is a signalling game with a seller and a buyer. The seller wants to sell a car to the buyer. The car can be of different qualities. We choose, by contrast to Akerlof, to introduce a finite number of qualities t_i, with i from 1 to n, and $t_i < t_{i+1}$ for any i between 1 and n-1. The seller's reservation price for a good of quality t_i is h_i, i from 1 to n, with $h_i < h_{i+1}$ for i from 1 to n-1. The seller sets a price for her car. The buyer observes the price and accepts or refuses the transaction. The buyer's reservation price for a good of quality t_i is H_i, i from 1 to n, with $H_i < H_{i+1}$ for any i between 1 and n-1. The buyer ignores the quality during the transaction, but has a prior probability distribution over the qualities, that is common knowledge of both players; the probability distribution assigns probability ρ_i to the quality t_i, with $0< \rho_i <1$ for any i from 1 to n and $\sum_{i=1}^{n}\rho_i =1$. It is assumed that $H_i > h_i$ for any i from 1 to n, in order to make profitable trade for both players possible. We also introduce the assumption:

$$\sum_{i=1}^{j}\frac{\rho_i H_i}{\sum_{k=1}^{j}\rho_k} < h_j \qquad \text{for j from 2 to n} \qquad \text{(a)}$$

and even the more restrictive assumption:

$$\sum_{i=f}^{j}\frac{\rho_i H_i}{\sum_{k=f}^{j}\rho_k} < h_j \qquad \text{for j from 2 to n and f from 1 to j-1.} \qquad \text{(b)}$$

Assumption (a) is the heart assumption of Akerlof's comment. Assumption (b) namely ensures that $H_i<h_{i+1}<H_{i+1}$ for any i from 1 to n-1.

Throughout the paper, we call this game the ***price model.*** It's symbolic representation (with two qualities) is given in figure 1.

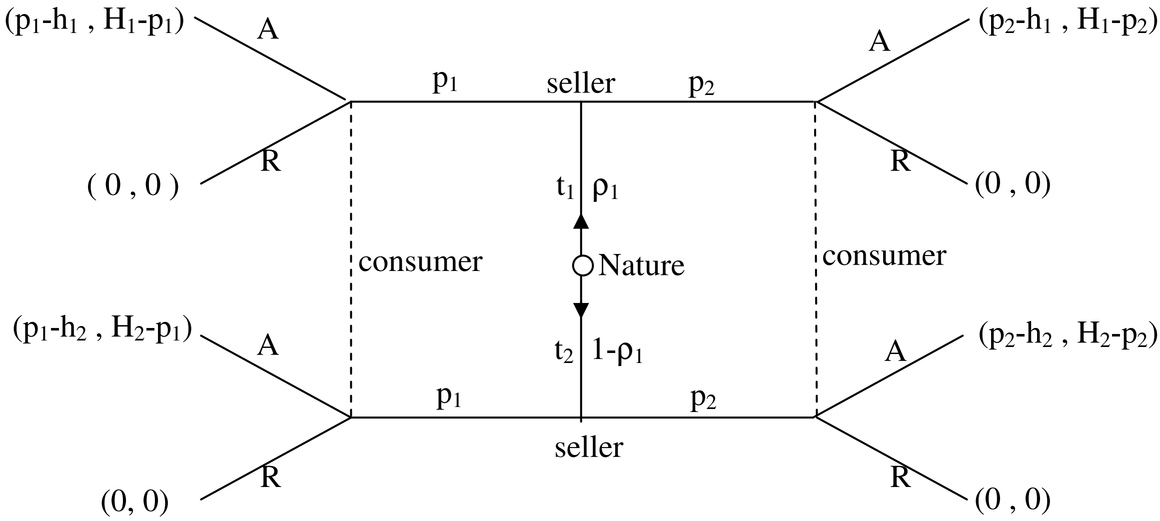

Figure 1

Legend: A and R mean that the consumer accepts (A) or refuses (R) the trade. The first, respectively the second coordinate of each vector of values, is the seller's, respectively the consumer's payoff.

Akerlof’s comment goes as follows:

If trade occurs, the car is sold at a unique price, regardless of its quality, because any type of seller wants to sell her car at the highest price. So imagine that the observed price is p, with $h_j \leq p < h_{j+1}$, j higher or equal to 2. Only qualities lower or equal to t_j can be sold at price p. It follows that the expected quality of the sold car is $\sum_{i=1}^{j} \frac{\rho_i t_i}{\sum_{k=1}^{j} \rho_k}$ and that the highest price the consumer accepts to pay is $\sum_{i=1}^{j} \frac{\rho_i H_i}{\sum_{k=1}^{j} \rho_k}$. Yet this price, by assumption (a), is lower than h_j and therefore lower than p. So trade will not occur at price p. As a consequence, trade can only occur at a price p lower than h_2. This price is necessarily assigned to quality t_1 and will be accepted, provided it is lower or equal to H_1. Therefore the worst quality throws all the other qualities out of the market.

The point we do not agree with in this comment is" that trade necessarily occurs at a unique price". In fact, many prices can coexist on the market, as soon as one introduces mixed Nash strategies. And this coexistence of prices will allow good qualities to be sold on the market, even in a context that satisfies the assumptions (a) and (b).

Throughout the paper $\pi_i(p_j)$ is the probability that the seller of type t_i plays p_j ; $q(p_j)$ is the probability that the consumer accepts price p_j.

Proposition 1

There exists an infinite number of mixed strategies PBE, in which the seller of type t_i plays the prices p_i^* and p_{i+1}^*, respectively with probabilities $1-\pi_i(p_{i+1}^*)$ and $\pi_i(p_{i+1}^*)$, with i from 1 to n-1; t_n plays the price p_n^* with probability 1.

$p_1^*=H_1$; $h_i < p_i^* < H_i$ for i from 2 to n (and therefore $p_i^* < p_{i+1}^*$ for i from 1 to n-1).

The buyer accepts p_1^* with probability 1 and accepts each price p_i^*, i from 2 to n, with probability $q(p_i^*)$.

$\pi_i(p_{i+1}^*)$, i from 1 to n-1, and $q(p_i^*)$, i from 1 to n, are defined by:

$$\pi_i(p_{i+1}^*) = \rho_{i+1}\, \pi_{i+1}(p_{i+1}^*)\, (H_{i+1}-p_{i+1}^*)/\ [\rho_i\, (p_{i+1}^* - H_i)] \quad (1)$$

$q(p_1^*)=1$

$$q(p_i^*)= (p_{i-1}^*-h_{i-1})q(p_{i-1}^*)/(p_i^*-h_{i-1}). \quad (2)$$

The buyer assigns each price p different from the equilibrium prices, with $H_{i-1} \leq p < H_i$, to t_{i-2}, for i from 3 to n, and each price p, with $H_1 < p < H_2$, to t_1. Hence he refuses the trade at each non equilibrium price higher than H_1. He accepts all the out of equilibrium prices lower than H_1.

Proof: see appendix 1

Given that both p_i* and p_{i+1}* are strictly higher than h_i for i from 1 to n-1 and that p_n is strictly higher than h_n, proposition 1 ensures that, as soon as the players are allowed to play mixed strategies, trade can occur with positive probabilities at different prices and each type's expected payoff can be positive at a PBE.

Does it make sense to work with mixed strategies in a price model? The answer is clearly yes. As a matter of fact, in the real world, it makes sense for a seller of type t_i to sometimes cheat, by playing the price p_{i+1}* played by the higher type t_{i+1}, given that this higher price is accepted from time to time. And it also makes sense to not always cheat, in that the consumer does not buy at price p_{i+1}* if his posterior beliefs on the types t_i and t_{i+1} are the prior ones. Symmetrically, it makes sense for the buyer to accept the price p_{i+1*} with a given probability, provided this price is mainly played by type t_{i+1}. And it is also optimal to not always accept p_{i+1*} , in order to not induce the type t_i to only play p_{i+1}*, a behaviour that would in turn preclude trade.

Let us focus on a special equilibrium, we call the ***mainly fair equilibrium PBE1***:

Let us assume that $H_i - H_{i-1} = K$ for i from 2 to n, $H_i - h_i = k$, with $k < K$, $\rho_i = 1/n$ for i from 1 to n. Let us look for the equilibrium which satisfies: $p_1^* = H_1$, $p_i^* = h_i + k/2 = H_i - k/2$ for i different from 1, t_n only plays p_n and t_i, i from 1 to n, only plays p_i and p_{i+1} with positive probability; we call this equilibrium mainly fair because both the buyer and the seller get the same payoff each time the seller of type t_i plays p_i* (except for i=1).

One obtains:

PBE1: mainly fair equilibrium

$\pi_{n-i}(p_{n-i+1}^*) = \delta(1+\delta^i)/(1+\delta)$ with $\delta = k/(2K-k)$ for i odd, i from 1 to n-1 (3a)

$$= \frac{k}{2K}\left(1+\left(\frac{k}{2K-k}\right)^i\right)$$

$\pi_{n-i}(p_{n-i+1}*) = \delta(1-\delta^i)/(1+\delta)$ for i even, i from 2 to n-2 (3b)

$$= \frac{k}{2K}(1-(\frac{k}{2K-k})^i)$$

$\pi_i(p_i*) = 1-\pi_i(p_{i+1}*)$ for i from 1 to n-1

$\pi_n(p_n*) = 1$

$q(p_1*)=1$

$q(p_i*)= 2k^{i-1}/(2K+k)^{i-1}$ for i from 2 to n. (4)

Proof: see appendix 2

Let us illustrate this equilibrium by a numerical example.

For $H_1=50$, $H_2=100$, $H_3=150$, $h_1=30$, $h_2=80$, $h_3=130$, $\rho_i=1/3$ for i from 1 to 3 and $p_1*=50$, $p_2*=90$ $p_3*=140$, one obtains $\pi_1(50)=13/16$, $\pi_1(90)=3/16$, $\pi_2(90)=3/4$, $\pi_2(140)=1/4$, $\pi_3(140)=1$, $q(50)=1$, $q(90)=1/3$ $q(140)=1/18= 0.056$.

In this equilibrium, the probability that each type of seller cheats (plays $p_{i+1}*$ instead of p_i* when she is of type t_i) rapidly converges to k/2K (1/5 if k=20 and K=50 like in the numerical example) when n becomes large.

The probability of accepting prices rapidly decreases with i, yet it is not equal to 0.

The limit of the mainly fair equilibrium is easy to understand by looking at figure 2:

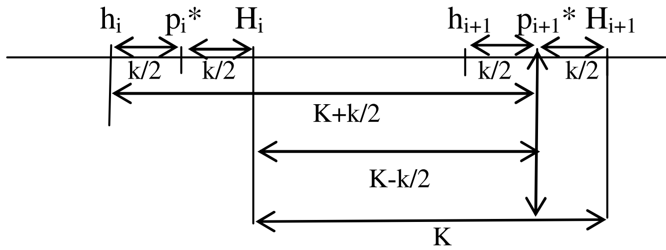

Figure 2

t_i has to be indifferent between p_i* and $p_{i+1}*$, which is only possible if what she gets with p_i*, i.e. k/2 multiplied by the probability that this price is accepted, is equal to what she gets when she plays $p_{i+1}*$ i.e. (K+k/2) multiplied by the probability that this price is accepted. One immediately gets the condition $q(p_{i+1}*)= kq(p_i*)/(2K+k)$ which is satisfied in (4).

Being indifferent between buying and not buying is only possible when buying leads to a 0 payoff. Call π the probability of cheating in the limit (i.e. when n goes to infinity). The payoff obtained with $p_{i+1}*$ is equal to k/2 when the consumer faces a seller who does not cheat, which happens with probability $\rho(1-\pi)$; the payoff is equal to –K+k/2 when he faces a seller who cheats, which happens with probability $\rho\pi$. It follows that his expected payoff is equal to 0 when $\pi = k/(2K)$.

3. The utility of additional strategic variables

Let us look at the following strategy profile:

PBE 2

$p_i^* = H_i$ for i from 1 to n

t_i plays p_i^* with probability 1;

$q(p_1^*)=1$

$$q(p_i^*) = \prod_{j=1}^{i-1} \frac{H_j - h_j}{H_{j+1} - h_j} \quad \text{for i from 2 to n}$$

The consumer accepts any price lower than H_1 and refuses any price higher than H_1 and different from H_j, j from 2 to n.

This strategy profile is a PBE .

Proof: it is enough to observe that the consumer, for each equilibrium price, is indifferent between buying and not buying, so he can choose both actions with positive probability. And the chosen probability is such that no type t_i is better off switching to the equilibrium price of another type t_j. Given that an out of equilibrium price can always be assigned to t_1, the proposed profile is a PBE path.

The proposed profile is a well known one, ***but in another model***. It is the Riley equilibrium of the experience good model, in which the seller both chooses a price and the quantity she wants to sell at this price. Enlarging the seller's strategy set is a widespread reaction to Akerlof's impossible trade result. So for example some papers introduce, in addition to the choice of a price, the choice of a date of transaction or the choice of a quantity. In the latter games, each type of seller chooses a couple (p,q) where p and q are respectively the price and the quantity she wants to sell at price p; usually one introduces an upper bound for q, equal to 1. The buyer, after observing the couple (p,q) accepts or refuses the transaction. When (p,q) is accepted, the seller's payoff, respectively the buyer's payoff, is defined by $(p - h_i)q$ and $(H_i-p)q$, when the good is of quality t_i, for i from 1 to n. In absence of trade, both payoffs are equal to 0. We call this model the ***price quantity model.***

If the price quantity model satisfies the same assumptions than the price model (especially assumptions (a) and (b) on the reservation prices and the prior probabilities), then the Riley equilibrium of the price quantity game is defined by:

Riley equilibrium

t_i plays (H_i,q_i^*), for i from 1 to n.

$q_1^*=1$

$$q_i^* = \prod_{j=1}^{i-1} \frac{H_j - h_j}{H_{j+1} - h_j}$$

The consumer accepts all the couples (H_i, q_i^*), i from 1 to n, and, for example, rejects all the out of equilibrium couples (p,q) with $p>H_1$ (by assigning p to t_1) and accepts all the couples (p,q) with $p \le H_1$.

It is immediate that the PBE 2 of the price model is the Riley equilibrium of the price quantity model. The probability of trade in the price model becomes the quantity bought by the consumer in the price quantity model. Therefore the only difference between both equilibria is that the seller's ***expected*** payoff (because trade occurs with probability q) in the price model becomes a ***sure*** payoff in the price quantity model, given that the quantity q is bought with probability 1. Yet this difference has no impact if the payoffs satisfy the Von Neuman Morgenstern axiomatic.

Given that the Riley equilibrium (when assumption (a) is satisfied) maximises the sellers' payoff in the price quantity model, it follows that the price is sufficient to get the same maximal payoff. In the numerical example ($H_1=50$, $h_1=30$, $H_2=100$, $h_2=80$, $H_3=150$, $h_3=130$ $\rho(t_i)==1/3$ for i from 1 to 3) PBE2 leads to the prices 50, 100 and 150 and to the probabilities (quantities) $q_1=1$, $q_2=2/7$ and $q_3=4/49$; the seller's maximal surplus in both the price and the price quantity models is therefore equal to 1340/147= 9.12.

Hence a simple additional strategic variable like the quantity is of no use for the seller, who can achieve the same maximal payoff without it.

So what is the added value of the quantity? Let us come back to the mainly fair equilibrium. It is easy to establish that the price quantity model leads to a PBE path, called ***fair equilibrium***, very close to PBE 1, in which t_1 plays $(H_1,1)$, t_i plays ($p_i^*= H_i-k/2$, $q_i^*= 2k^{i-1}/(2K+k)^{i-1}$) for i from 2 to n and the consumer accepts all these couples. The seller's payoff is again not affected by the switch from the price model to the price quantity model, given that the equilibrium quantities in the price quantity model are the probabilities of trade of one unit of good in the price model. But, by contrast to the Riley equilibrium, the fair and the mainly fair equilibria display two main differences.

The first difference is that the buyer's payoff is equal to 0 in the mainly fair equilibrium, whereas it is positive in the fair equilibrium. Indeed, in the price quantity model, p_i* is only played by t_i, hence the buyer gets q_i*$k/2= k^i/(2K+k)^{i-1}$ when he accepts the couple (p_i*, q_i*), i from 2 to n. In our numerical example (H_1=50, H_2=100, H_3=150, h_1=30, h_2=80, h_3=130, p_1=50, p_2=90, p_3=140, $\rho(t_i)$=1/3 for i from 1 to 3), the consumer's surplus becomes 10.1/3+10.1/18= 70/18, the seller's payoff being unchanged. By contrast, in the price model, p_i* is both played by t_i and t_{i-1}, in such a way that the buyer's expected payoff is null, which precisely allows him to accept the price with probability q_i*. Moreover, ***the nullity of the buyer's payoff holds in every PBE of the price model.***

Proposition 2

In every PBE of the price model in which each type of seller gets a positive payoff:

- each type of seller plays at most 3 prices with a positive probability;
- if t_i plays 3 prices p, p', and p", with p<p'<p", then p'=H_i;
- if t_i plays a price p different from H_i, then p is also played with positive probability by the adjacent type t_{i-1} or t_{i+1};
- at most 2n-1 different prices are played in a PBE path;
- the buyer's payoff is null.

In every PBE of the price model, the buyer's payoff is null.

Proof: see appendix 3

It follows that the additional quantity variable is able to increase the social surplus. In our numerical example, it is namely possible to show that the maximal social surplus in the price quantity model is obtained in the PBE path in which p_1=50, p_2=100, p_3=130, q_1=1, q_2=2/7,and q_3= 4/35; the social surplus is equal to 28/3. This surplus is higher than the maximal social surplus in the price model, given that it shrinks to the maximal surplus of the seller which is equal to the lower value 1340/147. An economist appreciates this difference, which is of course a positive point for the introduction of the additional variable.

Yet the second difference between the fair and the almost fair equilibrium is that by switching from the price model to the price quantity model, we loose the cheating phenomenon. As a matter of fact, the probabilities $\pi_i(p_{i+1}*)$ disappear in the switch, given that in the fair equilibrium t_i only plays the couple (p_i*,q_i*). Given that, for a high number of qualities, the probability of cheating goes to k/2K (= 1/5 in the numerical example), we loose this behaviour which consists, for each type except for the highest, to play the immediate higher price one fifth of the time. Is this loss a good or a bad point? Economists usually do not appreciate cheating because, usually, cheating is harmful for some players (here the consumer). But, nevertheless, cheating is, whether right-minded persons like it or not, one of the cornerstones of human behaviour; hence eliminating it through a structural change in the played game is perhaps not the only way to cope with it.

So, in the next sections, we approach the price model in a different way. We show that by switching to another equilibrium approach, one may keep the cheating phenomenon and simultaneously get a high social surplus.

4. Best-reply matching in normal form games

Let us turn to the best reply matching equilibria (BRM) introduced by Droste, Kosfeld & Voorneveld (2003). The definition of this concept is given for games in normal form. It is recalled hereby:

Definition (Droste & al. 2003)).

Let $G=(N, (S_i)_{i\in N}, \succ_i, i\in N)$ be a game. A mixed strategy p ($p\in P$) is a BRM equilibrium if for every player $i \in N$ and for every pure strategy $s_i\in S_i$, :

$$p_i(s_i)= \sum_{s_{-i}\in B_i^{-1}(s_i)} \frac{1}{\text{Card } B_i(s_{-i})} p_{-i}(s_{-i})$$

In a BRM equilibrium, the probability assigned to a pure strategy is linked to the number of times it is a best response to the strategies played by the opponents. So, for example, if player i' s opponents play s_{-i} with probability $p_{-i}(s_{-i})$, and if the set of i's best responses to s_{-i} is the subset of pure strategies $B_i(s_{-i})$, then each strategy of this subset is played with the probability $p_{-i}(s_{-i})$ divided by the cardinal of $B_i(s_{-i})$. This concept makes sense it that it displays a consistency which is not present in the mixed Nash equilibrium. Indeed, in a BRM equilibrium, the probability assigned to a pure strategy of player i is not the probability that justifies the other's strategies (as it is the case in the mixed Nash equilibrium) but it is relative to the number of times the strategy is a best response for player i. We agree with the authors that this consistency is very realistic, given that people, in general, are more concerned with the justification of their own behaviour than with the justification of the behaviour of others (which happens in mixed Nash equilibria) .[3]

Let us study the implication of such a criterion in the price model in normal form.

We first focus on a game with two qualities t_1 and t_2 and *only two prices* p_1 and p_2, with $h_1 < p_1 < H_1$, $h_2 < p_2 < H_2$ and $\rho_1 H_1 + \rho_2 H_2 < h_2$, in accordance with the assumptions introduced in section 2.

In this context, the best reply matching concept can be addressed by the use of table 1:

		q_1	q_2	q_3	q_4
		$A/p_1A/p_2$	$A/p_1R/p_2$	$R/p_1A/p_2$	$R/p_1R/p_2$
r_1	$p_1/t_1p_1/t_2$	B_2	B_2		$\mathbf{b_1}$
r_2	$p_1/t_1p_2/t_2$	B_2	$\mathbf{b_1}$		$\mathbf{b_1}$
r_3	$p_2/t_1p_1/t_2$		B_2		$\mathbf{b_1}$
r_4	$p_2/t_1p_2/t_2$	$\mathbf{b_1}$	B_2	$\mathbf{b_1}$	$\mathbf{b_1}$ B_2

Table 1

Legend of table 1 *(and the following tables):* ***b_1*** *means that the seller's strategy is a best reply to the buyer's strategy, B_2 means that the buyer's strategy is a best reply to the seller's strategy.*

Table 1 tells when a strategy is a best reply:

For example, $(p_1/t_1\ p_2/t_2)$ is a best response for player 1 each time player 2 plays $(R/p_1\ R/p_2)$ or $(A/p_1\ R/p_2)$. In the same way, for example, $(A/p_1\ A/p_2)$ is a best response for player 2 each time player 1 plays $(p_1/t_1\ p_1/t_2)$ or $(p_1/t_1\ p_2/t_2)$. $(R/p_1\ A/p_2)$ is never a best response for player 2.

r_1, r_2, r_3 and r_4 are the probabilities assigned to $(p_1/t_1p_1/t_2)$, $(p_1/t_1p_2/t_2)$, $(p_2/t_1p_1/t_2)$ and $(p_2/t_1p_2/t_2)$; q_1, q_2, q_3 and q_4 are the probabilities assigned to $(A/p_1A/p_2)$, $(A/p_1R/p_2)$,

[3] One observes that this consistency is compatible with Nash's consistency when the Nash equilibria are strict. In a strict Nash equilibrium player i's strategy s_i* is the only best reply to the other players' strategies s_{-i}*. It follows that, according to the BRM logic, s_i* has to be played with the probability of play of s_{-i}*, i.e. 1. Hence a strict Nash equilibrium is also a BRM equilibrium.

$(R/p_1A/p_2)$ and $(R/p_1R/p_2)$.It follows that $(p_1/t_1p_1/t_2)$ will be played with probability $q_4/4$, because $(R/p_1R/p_2)$ is supposed to be played with probability q_4 and there are 4 best responses for player 1 to this strategy. Hence $r_1= q_4/4$.

In a similar way we get the following equations:

$q_1 = r_1/2 + r_2$

$r_2= q_2+ q_4/4$	$q_2 = r_1/2 + r_3 + r_4/2$
$r_3= q_4/4$	$q_3 = 0$
$r_4= q_1+ q_3 + q_4/4$	$q_4 = r_4/2$
$r_1+r_2+r_3+r_4=1$	$q_1+q_2+q_3+q_4=1$

This system leads to

$r_1= 2/33$, $r_2=13/33$, $r_3=2/33$, $r_4= 16/33$

$q_1= 14/33$, $q_2=11/33$, $q_3=0$, $q_4= 8/33$

The Kuhn equivalent behavioural strategies are:

$\pi_1(p_1) = r_1+r_2 = 15/33$	$\pi_1(p_2) = r_3+r_4 = 18/33$
$\pi_2(p_1) = r_1+r_3 = 4/33$	$\pi_2(p_2) = r_2+r_4 = 29/33$

$q(p_1) = q_1+q_2= 25/33$

$q(p_2) = q_1+q_3= 14/33$

Let us comment the values of $\pi_1(p_1)$, $\pi_1(p_2)$, $\pi_2(p_1)$, $\pi_2(p_2)$, $q(p_1)$ and $q(p_2)$. One observes that the probabilities have been obtained without taking into account the exact values of the payoffs of the players. Hence each time $h_1<p_1<H_1$, $h_2<p_2<H_2$ and $\rho_1H_1+\rho_2H_2< h_2$, we will get the same BRM equilibrium. This is not a strange property given that pure Nash equilibria also share it. Yet mixed Nash equilibria do not share it (the values of the probabilities depend on the values of the parameters of the games). Hence it is difficult to compare the unique BRM equilibrium to the mixed PBE of the price model (reduced to two prices p_1 and p_2), which differ according to the values of p_1 and p_2. But we can nevertheless make the following comments:

First, like in the PBE of proposition 1, t_1 plays both prices p_1 and p_2 and the consumer both accepts and refuses p_2. t_2 almost only plays p_2 and the buyer almost always accepts p_1, two results that are close to the ones obtained in the PBE equilibrium. But, by contrast to the PBE, whose equilibrium probabilities depends on the exact values of p_1 and p_2 and the other parameters of the game, in the BRM equilibrium t_1 plays each price among half of time

(15/33 and 18/33 are close to ½) and the buyer accepts p_2 with a probability close to ½ also (14/33), regardless of the exact values of the parameters.

The same exercise can be done for a higher number of types. For example let us turn to three types, t_1, t_2 and t_3 and 3 prices p_1, p_2 and p_3, with $h_1<p_1<H_1$, $h_2<p_2<H_2$, $h_3<p_3<H_3$, $(\rho_1H_1+\rho_2H_2)/(\rho_1+\rho_2)< h_2$ and $\rho_1H_1+\rho_2H_2+\rho_3H_3< h_3$. We also assume, like in the numerical almost fair equilibrium, that $p_2<(\rho_1H_1+\rho_3H_3)/(\rho_1+\rho_3)$. The best-reply table is table 2.

Writing $r= q_8/27$, the system of equations becomes:

$r_i= r= q_8/27$ for i from 1 to 3, from 5 to 6, from 10 to 11, from 13 to 16, from 18 to 24, from 26 to 27 included.

$r_4= r_7=r_8=r_9= q_8/27 + q_4/4$

$r_{12}= q_8/27 + q_6/2$

$r_{17}= q_8/27 + q_6/2 + q_2$

$r_{25}= q_8/27 + q_1+ q_3+ q_5 + q_7$

$\sum_{i=1}^{27} r_i =1$, $\sum_{i=1}^{8} q_i =1$

$q_1= 5r/2 + 3r_4/2$

$q_2= 8r + 2r_4 + r_{25} /4$

$q_3= 9r/4 + r_{12}/2 +r_{17}/2$

$q_4= 17r/4 + r_4/2 + r_{12}/2 +r_{25}/4$

$q_5= r/4$

$q_6= 9r/4 + r_{25}/4$

$q_7= r_{17}/2$

$q_8= r/2 + r_{25}/4$

One obtains:

$r= 1/264$, $r_4=r_7=r_8=r_9 = 97/2112$, $r_{12}= 123/2112$, $r_{17}= 593/2112$, $r_{25}= 106/264$

$q_1=331/4224$, $q_2= 470/2112$, $q_3= 752/4224$, $q_4= 712/4224$, $q_5= 1/1056$, $q_6= 115/1056$, $q_7= 593/4224$ $q_8= 108/1056$

The Kuhn equivalent behavioural strategies are:

$\pi_1(p_1) = r_1+r_2+r_3+r_4+r_5+r_6+r_7+r_8+r_9= 428/2112= 0.20$

$\pi_1(p_2) = r_{10}+r_{11}+r_{12}+r_{13}+r_{14}+r_{15}+r_{16}+r_{17}+r_{18}= 772/2112= 0.37$

$\pi_1(p_3) = r_{19}+r_{20}+r_{21}+r_{22}+r_{23}+r_{24}+r_{25}+r_{26}+r_{27}= 114/264= 0.43$

		q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8
		$A/p_1A/p_2A/p_3$	AAR	ARA	ARR	RAA	RAR	RRA	RRR
r_1	$p_1/t_1p_1/t_2p_1/t_3$	B_2	B_2	B_2	B_2				$\mathbf{b_1}$
r_2	$p_1p_1p_2$	B_2	B_2						$\mathbf{b_1}$
r_3	$p_1p_2p_1$	B_2	B_2						$\mathbf{b_1}$
r_4	$p_1p_2p_2$	B_2	B_2		$\mathbf{b_1}$				$\mathbf{b_1}$
r_5	$p_1p_1p_3$	B_2		B_2					$\mathbf{b_1}$
r_6	$p_1p_3p_1$		B_2		B_2				$\mathbf{b_1}$
r_7	$p_1p_3p_3$		B_2		$\mathbf{b_1}$ B_2				$\mathbf{b_1}$
r_8	$p_1p_2p_3$	B_2			$\mathbf{b_1}$				$\mathbf{b_1}$
r_9	$p_1p_3p_2$		B_2		$\mathbf{b_1}$				$\mathbf{b_1}$
r_{10}	$p_2p_1p_1$			B_2	B_2				$\mathbf{b_1}$
r_{11}	$p_2p_1p_2$	B_2	B_2						$\mathbf{b_1}$
r_{12}	$p_2p_2p_1$			B_2	B_2		$\mathbf{b_1}$		$\mathbf{b_1}$
r_{13}	$p_2p_2p_2$	B_2	B_2			B_2	B_2		$\mathbf{b_1}$
r_{14}	$p_2p_1p_3$			B_2					$\mathbf{b_1}$
r_{15}	$p_2p_3p_1$				B_2				$\mathbf{b_1}$
r_{16}	$p_2p_3p_3$				B_2				$\mathbf{b_1}$ B_2
r_{17}	$p_2p_2p_3$		$\mathbf{b_1}$	B_2			$\mathbf{b_1}$	B_2	$\mathbf{b_1}$
r_{18}	$p_2p_3p_2$		B_2				B_2		$\mathbf{b_1}$
r_{19}	$p_3p_1p_1$		B_2		B_2				$\mathbf{b_1}$
r_{20}	$p_3p_1p_2$		B_2						$\mathbf{b_1}$
r_{21}	$p_3p_2p_1$		B_2						$\mathbf{b_1}$
r_{22}	$p_3p_2p_2$		B_2				B_2		$\mathbf{b_1}$
r_{23}	$p_3p_1p_3$		B_2		B_2				$\mathbf{b_1}$
r_{24}	$p_3p_3p_1$		B_2		B_2				$\mathbf{b_1}$
r_{25}	$p_3p_3p_3$	$\mathbf{b_1}$	B_2	$\mathbf{b_1}$	B_2	$\mathbf{b_1}$	B_2	$\mathbf{b_1}$	$\mathbf{b_1}$ B_2
r_{26}	$p_3p_2p_3$		B_2				B_2		$\mathbf{b_1}$
r_{27}	$p_3p_3p_2$		B_2				B_2		$\mathbf{b_1}$

Table 2

$\pi_2(p_1) = r_1+r_2+r_5+r_{10}+r_{11}+r_{14}+r_{19}+r_{20}+r_{23}= 9/264 = 0.03$

$\pi_2(p_2) = r_3+r_4+r_8 +r_{12} +r_{13} +r_{17} +r_{21}+r_{22} +r_{26}= 950/2112= 0.45$

$\pi_2(p_3) = r_6+r_7 +r_9 +r_{15} +r_{16} +r_{18} +r_{24} +r_{25} +r_{27}= 1090/2112= 0.52$

$\pi_3(p_1) = r_1+r_3 +r_6+r_{10}+r_{12} +r_{15}+r_{19}+r_{21}+r_{24}= 187/2112= 0.09$

$\pi_3(p_2) = r_2+r_4+r_9 +r_{11} +r_{13} +r_{18} +r_{20}+r_{22} +r_{27}= 250/2112= 0.12$

$\pi_3(p_3) = r_5+r_7 +r_8 +r_{14} +r_{16} +r_{17} +r_{23} +r_{25} +r_{26}= 1675/2112= 0.79$

$q(p_1)=q_1 +q_2 +q_3 +q_4 = 2735/4224= 0.65$

$q(p_2)= q_1 +q_2 +q_5 +q_6 = 1735/4224= 0.41$

$q(p_3)= q_1 +q_3 +q_5 +q_7 = 840/2112 = 0.40$

These results lead us to observations that will generalize:

Proposition 3

The BRM equilibrium displays some common points with the PBE equilibria (of proposition 1) obtained for 3 prices, but also two main differences. The common points are first that t_i is the type who plays p_i with the highest probability, second that all prices are accepted with a significant probability. ***The main difference is that t_1 does not only play p_1 and p_2 but she also plays p_3 with a significant probability***. More generally, in a model with n types of seller, t_i plays all the prices p_i, $p_{i+1},\ldots,p_n$ with a significant positive probability. ***This is impossible in any PBE with a positive payoff for the seller (see proposition 2).*** This difference is linked to another difference. In the PBE equilibria, the probabilities of accepting the price p_i geometrically decreases in i. This fact is no longer true with the BRM concept.

5. Best-reply matching in normal form games without weakly dominated strategies

Let us come back to the BRM equilibrium in the game with only two types. $q(p_1)$ is different from 1 because the consumer is indifferent between accepting and refusing p_1 when p_1 is not played by any type of seller. Yet, given that p_1 is lower than H_1, a buyer's strategy that refuses p_1 is weakly dominated by the strategy that accepts p_1, the behaviour after p_2 being equal. In the same way, $\pi_2(p_1)$ is different from 0, because p_1 can lead to a 0 payoff each time the consumer refuses both prices. Yet a seller's strategy that leads to the play of p_1 when the seller is of type t_2 is weakly dominated by the strategy that leads to the play of p_2 when the player is of type t_2, the action by type t_1 being equal. So, in order to eliminate these rather non natural probabilities, let us turn to the game in which t_2 always plays p_2 and the consumer always accepts p_1. The best-reply table is table 3:

		q_1	q_2
		$A/p_1A/p_2$	$A/p_1R/p_2$
r_1	$p_1/t_1p_2/t_2$	B_2	$\mathbf{b_1}$
$\mathbf{r_2}$	$\mathbf{p_2/t_1p_2/t_2}$	$\mathbf{b_1}$	B_2

Table 3

The system of equations becomes:

$r_1= q_2 \quad r_2 =q_1 \quad q_1=r_1 \quad q_2=r_2$.

Therefore $r_1= r_2=q_1 =q_2= ½$ and the Kuhn behavioural equivalent strategies become:

$\pi_1(p_1) = r_1=0.5$, $\pi_1(p_2) = r_2=0.5$, $\pi_2(p_2) = 1$

$q(p_1) = 1$, $q(p_2) = q_1= 0.5$.

Hence t_1 plays both prices with probability ½ and the consumer accepts the high price with probability ½ , a result which is close to the one obtained in the price model with the weakly dominated strategies. It follows that, in this game, the elimination of weakly dominated strategies has almost no impact on the BRM equilibrium.

In the three type case, with $h_1<p_1<H_1$, $h_2<p_2<H_2$, $h_3<p_3<H_3$, $(\rho_1H_1+\rho_2H_2)/ (\rho_1+\rho_2)< h_2$ and $\rho_1H_1+\rho_2H_2+\rho_3H_3< h_3$, each seller's strategy in which she plays p_1 if she is of type t_2 is weakly dominated by the strategy in which she plays p_2 when she is of type t_2, the actions played by types t_1 and t_3 being equal. Similarly, each seller's strategy in which she plays p_1 or p_2 if she is of type t_3 is weakly dominated by the strategy in which she plays p_3 when she is of type t_3, the actions played by types t_1 and t_2 being equal. Similarly a strategy such that the consumer refuses p_1 is weakly dominated by the strategy in which he accepts p_1, the behaviour after p_2 and p_3 being equal. Hence eliminating weakly dominated strategies leads to the table 4.

The system of equations becomes:

$r_1= r_2= q_4/2$, $r_3=q_2$, $r_4=r_5=0$, $r_6=q_1+q_3$

$q_1=r_1$, $q_2=r_2/2+r_5+r_6/2$, $q_3=r_3$, $q_4= r_2/2+r_4+r_6/2$

It follows that $r_1=r_2=1/7$, $r_3=2/7$, $r_4=r_5=0$, $r_6=3/7$, $q_1= 1/7$, $q_2=q_3= q_4= 2/7$.

The Kuhn equivalent behavioural strategies are:

$\pi_1(p_1) = 2/7$, $\pi_1(p_2) = 2/7$, $\pi_1(p_3) = 3/7$

$\pi_2(p_2) = 3/7$, $\pi_2(p_3) = 4/7$, $\pi_3(p_3) = 1$

$q(p_1)= 1$, $q(p_2)= 3/7$, $q(p_3)= 3/7$.

		q_1	q_2	q_3	q_4
		$A/p_1A/p_2A/p_3$	$A/p_1A/p_2R/p_3$	$A/p_1R/p_2A/p_3$	$A/p_1R/p_2R/p_3$
r_1	$p_1/t_1 p_2/t_2 p_3/t_3$	B_2			$\mathbf{b_1}$
r_2	$p_1/t_1 p_3/t_2 p_3/t_3$		B_2		$\mathbf{b_1}$ B_2
r_3	$p_2/t_1 p_2/t_2 p_3/t_3$		$\mathbf{b_1}$	B_2	
r_4	$p_2/t_1 p_3/t_2 p_3/t_3$				B_2
r_5	$p_3/t_1 p_2/t_2 p_3/t_3$		B_2		
r_6	$p_3/t_1 p_3/t_2 p_3/t_3$	$\mathbf{b_1}$	B_2	$\mathbf{b_1}$	B_2

Table 4

Except for $\pi_1(p_1)$ and $\pi_1(p_2)$ which are respectively significantly higher and lower than their values in the original game (namely because accepting p_1 with probability 1 increases its play by t_1), the results are again similar to the one obtained without the elimination of the mixed strategies. Therefore the conclusions do not significantly differ from the ones obtained in the preceding approach.

6. Best-reply matching in extensive form games without weakly dominated strategies

Unfortunately, whereas Nash equilibria select the same issues in both the normal or the extensive form of a game, the BRM equilibrium concept does not select the same issues in both representative forms of a game.

Let us be more precise. Droste & al 's (2003) BRM definition is given for the normal form only. So let us first propose a natural extension of their definition to the extensive form of a signalling game:

Extension of the BRM equilibrium concept to extensive form signalling games

Let G be a finite signalling game in extensive form. Player 1 can be of n types and sells at most M messages. Player 2 observes each message m and responds with an action r out of R(m), the finite set of actions available at message m. A behavioural strategy profile $((\pi_{t_1}(.)...\pi_{t_n}(.), \pi_{2m_1}(.)...\pi_{2m_M}(.))$ is a BRM equilibrium if:

-for every type t_i of player 1, and every message m_i available to type t_i,

$$\pi_{t_i}(m_i) = \sum_{r \in B_{t_i}^{-1}(m_i)} \frac{1}{\text{Card } B_{t_i}(r)} \prod_{j=1}^{M} \pi_{2m_j}(r_j)$$

where $r = (r_1, \ldots r_M)$ is a profile of actions played by player 2, and $B_{t_i}(r)$ is the set of best responses of type t_i to the profile r.

- after each message m_k, for every action r_{m_k} available after m_k:

$$\pi_{2m_k}(r_{m_k}) = \sum_{m \in B_{2m_k}^{-1}(r_{m_k})} \frac{1}{\text{Card } B_{2m_k}(m)} \prod_{i=1}^{n} \pi_{t_i}(m_i)$$

where $m=(m_1 \ldots m_n)$ is the profile of messages sent by the n types of player 1 and $B_{2m_k}(m)$ is the subset of player 2's best response to the profile m after observing m_k.

Let us apply this definition to the price model with two qualities t_1 and t_2 and two prices p_1 and p_2, with $h_1<p_1<H_1$, $h_2<p_2<H_2$ and $\rho_1 H_1+\rho_2 H_2<h_2$. We also eliminate the weakly dominated strategies, given that their elimination seems not to have a strong impact on the results. Hence t_2 can only play p_2 and the consumer can only accept p_1. It follows that the BRM equilibrium logic in the extensive form of the game leads to:

$\pi_1(p_1)= 1-q(p_2)$

$\pi_1(p_2)= q(p_2)$ given that t_1's best response is p_2 each time the consumer accepts p_2, and p_1 in the remaining case.

$\pi_2(p_2)= 1$

$q(p_1)= 1$

$q(p_2) = 1-\pi_1(p_2)$ given that player 2's best response when he observes p_2 is to accept p_2 if and only if t_1 plays p_1.

It immediately follows that:

$\pi_1(p_2)= \pi_1(p_1)= ½$, $\pi_2(p_2)= 1$, $q(p_1)=1$ and $q(p_2)=1/2$.

Hence, in the two type game, we get exactly the same result regardless of the representative form of the game.

Unfortunately, this equality of results does not generalize.

So let us first turn to the game with three types, t_1, t_2 and t_3, with $h_1<p_1<H_1$, $h_2<p_2<H_2$, $h_3<p_3<H_3$, $(\rho_1H_1+\rho_2H_2)/(\rho_1+\rho_2)< h_2$, $\rho_1H_1+\rho_2H_2+\rho_3H_3< h_3$. We also eliminate the weakly dominated strategies, so that we can write: $q(p_1)=1$, $\pi_3(p_3)=1$ and $\pi_2(p_1)=0$. The studied game is the one given in the 4 figures 3.

The system of equations becomes:

$\pi_1(p_3)=q(p_3)$

$\pi_1(p_2)= (1-q(p_3))q(p_2)$

given that t_1's best response is p_3 each time the consumer accepts p_3 and it is p_2 each time the consumer refuses p_3 but accepts p_2. With the remaining probability (not written here) t_1 plays p_1.

$\pi_2(p_3)= q(p_3) + (1-q(p_3))(1-q(p_2))/2$

given that t_2's best reply is to play p_3 each time p_3 is accepted and also each time both p_3 and p_2 are refused. In the latter case, both p_2 and p_3 are best replies, which explains the division by 2. t_2 plays p_2 with the remaining probability (not written here).

$\pi_3(p_3)= 1$

$q(p_1)= 1$

$q(p_2)= (1- \pi_1(p_2))\, \pi_2(p_2)+(1-\pi_1(p_2))(1-\pi_2(p_2))/2$

because accepting p_2 is optimal if only t_2 plays p_2 or if neither t_1 nor t_2 play p_2. In the latter case, player 2 can also refuses p_2, which explains the division by 2. The consumer refuses p_2 with the remaining probability.

$q(p_3)= (1-\pi_1(p_3))(1-\pi_2(p_3))$

because accepting p_3 is optimal only if t_1 and t_2 do not play p_3. The consumer rejects p_3 with the remaining probability.

Solving the system of equations leads to:

$\pi_1(p_1)= \pi_1(p_2)=\pi_1(p_3)=1/3$, $\pi_2(p_2)= \pi_2(p_3)=1/2$, $\pi_3(p_3)=1$

$q(p_1)=1$, $q(p_2)=1/2$ and $q(p_3)=1/3$.

Let us comment this result.

First, even if the seller's behaviour is not far removed from the one in the normal form game (2/7, 2/7, 3/7 become 1/3,1/3,1/3 and 3/7 becomes ½), ***the results obtained in the extensive form are different from the ones obtained in the normal form.*** Let us give some insights into why the results are different.

In the extensive form game, we both have:

$\pi_1(p_2)= (1-q(p_3))q(p_2)$ (given that t_1's best response is p_2 each time the consumer refuses p_3 but accepts p_2)

and $\pi_2(p_3)= q(p_3) + (1-q(p_3))(1-q(p_2))/2$

(given that t_2's best reply is to play p_3 each time p_3 is accepted and also each time both p_3 and p_2 are rejected).

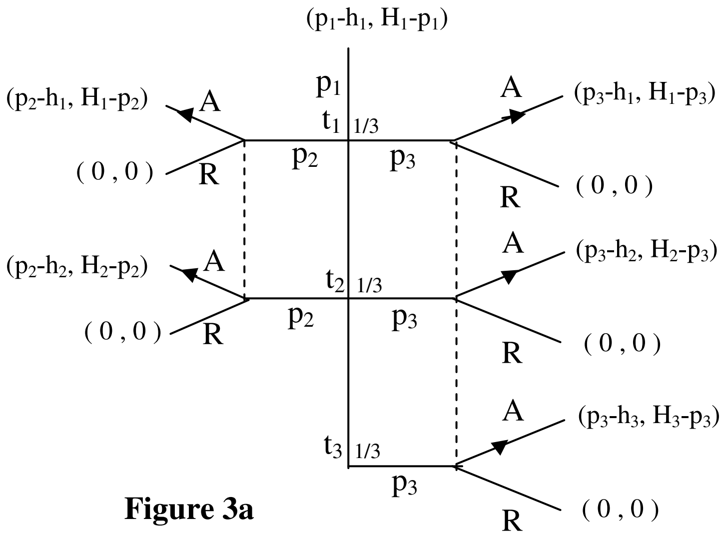

Figure 3a

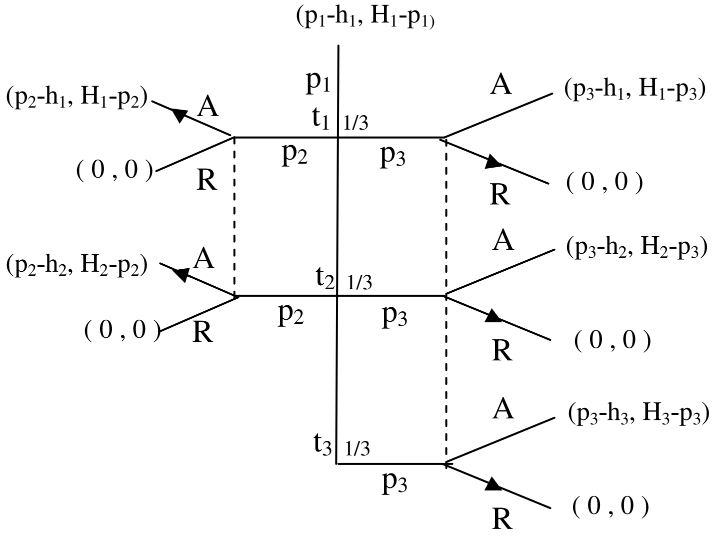

Figure 3b

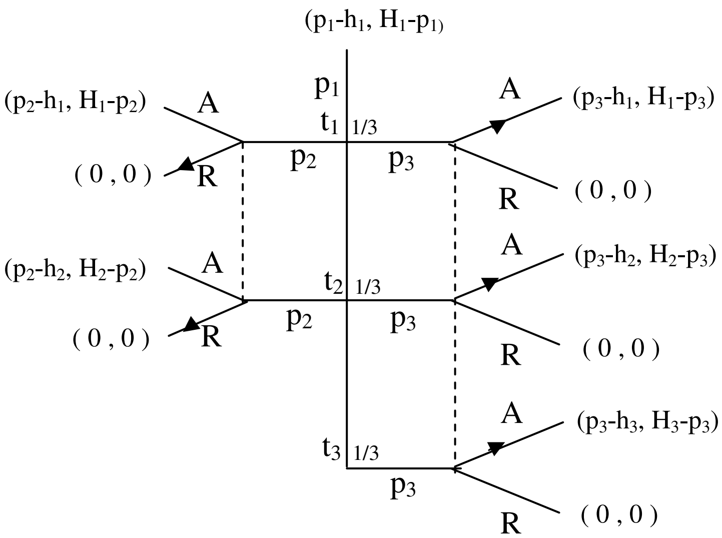

Figure 3c

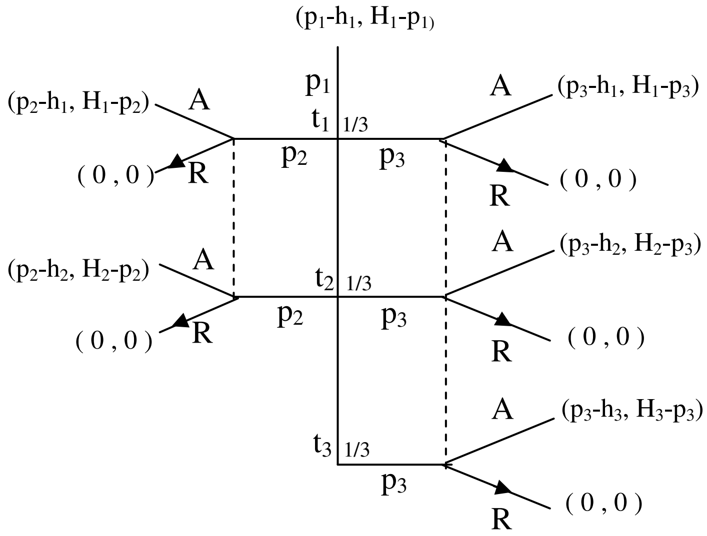

Figure 3d

In other words, p_2 is optimal for t_1 in the strategy configuration of figure 3b, and p_3 is optimal for t_2 in the strategy configurations of figures 3a, 3c and 3d.. Hence there is no configuration such that simultaneously p_2 is optimal for t_1 and p_3 is optimal for t_2. Yet this does not prevent p_2 from being optimal for t_1 in some circumstances and p_3 from being optimal for t_2 in other circumstances. It follows that the BRM concept in extensive form clearly uses the ***decentralization of the decisions taken by t_1 and t_2***, a decentralization which is possible in the extensive form approach.

This decentralization is impossible in the normal form approach. So we observe in table 4 that $p_2/t_1p_3/t_2\ p_3/t_3$ is never a best response given that there is no configuration of player 2's responses such t_1 and t_2 are simultaneously best off by playing p_2 and p_3 respectively. The normal form links the actions taken at each decision node of the seller and therefore looks for configurations of actions by the consumer that justify a profile of decisions of the seller (one at each information set). Hence the logic is different and this explains that both approaches do not lead to the same results.

Second, the obtained result is worth of interest in that **the obtained behaviours are elementary**: t_1 can play 3 prices and plays each of them with probability 1/3, t_2 can play 2 prices and plays each of them with probability ½, t_3 can only play one price and of course plays it with probability 1; the buyer accepts p_1 –which can only be played by t_1- with probability 1, he accepts p_2- which can be played by 2 types- with probability ½, and he accepts p_3 –which can be played by 3 types- with probability 1/3. We prove in the next section that this behaviour can be generalized.

7. Best reply matching in extensive form: a simple behaviour rule that should be experimentally tested

In this section we prove that the above behaviour generalizes as soon as one smoothly changes the behaviour of some players when they are indifferent between several best responses. Droste & al.(2003) tell in their paper that there is no real motivation to assign to each best response the same probability (by dividing by the cardinal of the subset of best responses).

Let us turn to the general case with n types, after elimination of the trivial weakly dominated strategies. So we focus on a game with n types, n prices p_1, p_2, ..p_n, with $h_i<p_i<H_i$, i from 1 to n, such that the consumer is strictly better off accepting p_i if only t_i plays p_i and is indifferent between accepting and refusing p_i only if nobody (i.e. no type lower or equal to t_i) plays p_i. In this latter case, ***we now suppose that, instead of accepting and refusing p_i with the probability of the event "no type lower or equal to t_i plays p_i" divided by 2, the consumer accepts p_i only with the probability of this event divided by i.*** Given that i is the cardinal of the set of types that can play p_i, we introduce in some way a kind of risk aversion that grows with higher prices. This is not a silly assumption even if we admit that we only introduce it in order to get the generalization of the result obtained in the three type case.

The system of equations in the general case becomes:

$\pi_1(p_n)=q(p_n)$

$\pi_1(p_i)= q(p_i)\prod_{j=i+1}^{n}(1-q(p_j))$ for i from 2 to n-1

$\pi_1(p_1)= 1- \sum_{i=2}^{n}\pi_1(p_i)$

$\pi_i(p_n)= q(p_n) +[\prod_{j=i}^{n}(1-q(p_j))]/ (n-i+1)$ for i from 2 to n-1

$\pi_i(p_k)= q(p_k) \prod_{j=k+1}^{n}(1-q(p_j)) +[\prod_{j=i}^{n}(1-q(p_j))]/ (n-i+1)$ for i from 2 to n-1 and k from i+1 to n-1

$\pi_i(p_i)= 1- \sum_{j=i+1}^{n}\pi_i(p_j)$

$\pi_n(p_n)= 1$

$q(p_1)= 1$

$q(p_i)= \pi_i(p_i) \prod_{j=1}^{i-1}(1-\pi_j(p_i))+ [\prod_{j=1}^{i}(1-\pi_j(p_i))] / i$ for i from 2 to n

It is easy to check that a solution for this system of equations is given by:

Proposition 4

In the n type case, the BRM behaviour is given by:

$\pi_i(p_j)= 1/(n-i+1)$ for i from 1 to n and j from i to n.

$q(p_j)= 1/j$ for j from 1 to n.

In other words, each type plays each available price with the same probability and the consumer accepts each price with the probability 1 divided by the number of types who can play this price. It is difficult to find a more easy behaviour, that displays the same amount of consistency. To our mind it would be worth testing experimentally if such a behaviour can be adopted by real players.

8. Conclusion: best-reply matching and social surplus

Let us conclude by observing that ***the preceding behaviour rule is not only simple and consistent but it can lead to positive payoffs for both the consumer and the seller, at least if the number of types is low. Moreover, the social surplus can be higher than the highest PBE social surplus in the price quantity model.***

Proposition 5

In the price model with two types and two prices p_1 and p_2, with $h_1<p_1<H_1$, $h_2<p_2<H_2$, and $\rho_1 H_1+\rho_2 H_2<h_2$, ***best reply matching can lead to positive payoffs for both the consumer and the seller. Moreover the social surplus can be higher than the highest PBE social surplus in the price quantity model and therefore also in the price model.***

To prove this proposition, we first observe that the highest social surplus achievable with a PBE in the price quantity model is obtained with $p_1= H_1$, $p_2=h_2$, $q_1=1$, $q_2= (H_1-h_1)/(h_2-h_1)$. It leads to the social surplus $\rho_1(H_1-h_1)+\rho_2 (H_2-h_2)(H_1-h_1)/(h_2-h_1)$.[4]

In the price model, in which the consumer's surplus is necessary null (cf. proposition 2), the highest social surplus limits to the highest seller's payoff, which is equal to the lower amount $\rho_1(H_1-h_1)+\rho_2(H_2-h_2)(H_1-h_1)/(H_2-h_1)$.

[4] This result derives from a maximization program.

Now let us look for the BRM equilibrium in the game with 2 prices, with $h_1<p_1<H_1$, $h_2<p_2<H_2$ and $\rho_1H_1+\rho_2H_2< h_2$, after elimination of the weakly dominated strategies (hence t_2 plays p_2 with probability 1 and the buyer accepts p_1 with probability 1). We know that in this case the normal form approach and the extensive form approach of BRM lead to the same result, i.e. t_1 plays p_1 and p_2 with probability ½ and the buyer accepts p_2 with probability ½. It follows that the surplus of the seller is $\rho_1[(p_1-h_1)1/2 + (p_2-h_1)1/2.1/2]+ \rho_2 (p_2-h_2)1/2$. The consumer's surplus is equal to $\rho_1[(H_1-p_1)1/2 + (H_1-p_2)1/2.1/2]+ \rho_2 (H_2-p_2)1/2$. So, first, the total surplus is equal to $\rho_1(H_1-h_1)3/4 + \rho_2 (H_2-h_2)\ 1/2$.

Take the values $H_1=50$, $h_1=49$, $H_2=70$, $h_2=61$, $\rho_1=\rho_2=0.5$, p_1 very close to 50[5] and $p_2=62$.

It is easy to check that the assumptions on the parameters are satisfied, that the highest PBE social surplus in the price quantity model is 3.5/4, that it is equal to 5/7 in the price model, and that the highest PBE consumer surplus is 1.5/4 in the price quantity model (it is null in the price model); the highest PBE seller payoff is 5/7. By contrast, the BRM social surplus is equal to 10.5/4, i.e. 3 times the maximal PBE social surplus of the price quantity model. The maximal consumer surplus for a p_1 close to 50 is obtained for p_2 very close to 61 and is therefore equal to 3.5/4, which is much higher than the maximal consumer surplus in the price quantity model. The highest BRM seller payoff is obtained for p_1 very close to H_1 and p_2 very close to H_2 and it is equal to 20.5/4 (the surplus of the consumer being negative in this case).

Moreover it is easy to find values of p_1 and p_2 that lead to positive payoffs for both players. For example, for p_1 very close to 50 and $p_2=62$, the consumer surplus is equal to 2/4 and the seller surplus is equal to 8.5/4. Both payoffs are not only positive, but they are both higher than the maximal consumer and seller surplus in a PBE in both the price and the price quantity model.

So we can conclude as follows: given that cheating is allowed in the BRM approach (t_1 cheats half of time), cheating does not necessarily lead to bad payoffs as soon as one gives up the Nash consistency approach. With the alternative consistency approach conveyed by the BRM concept, cheating can be socially efficient in that each type of seller as well as the consumer can get a positive payoff. So the bad quality does do not necessarily throw out the

[5] We choose p_1 very close (approximately equal) to H_1 in order to show that the result is not linked to the fact that p_1 can be chosen lower than H_1 in the BRM approach whereas it has to be higher or equal to H_1 in any PBE.

good quality out of the market even in a simple price model, and everybody can get a positive payoff.

References

Akerlof G.A. 1970. The market for "lemons": quality uncertainty and the market mechanism. The Quarterly Journal of Economics, Vol 84, Issue 3, 488-500.

Droste E., Kosfeld M., Voorneveld M. 2003. Best-reply matching in games. Mathematical Social Sciences 46, 291-309.

Appendix 1

(2) implies that t_i, with i from 1 to n-1, is indifferent between p_i^* and p_{i+1}^* :

$(p_{i+1}^*-h_i).q(p_{i+1}^*)= (p_i^*-h_i)q(p_i^*)$ (5)

It follows from (5), and from the definition of p_i^*, that $q(p_i^*)$ decreases in i .

Let us prove that, for i from 2 to n-1, t_i prefers p_i^* and p_{i+1}^* to any p_j^*, with j higher than i+1:

$(p_{j+1}^*-h_j).q(p_{j+1}^*)= (p_j^*-h_j)q(p_j^*)$ for j from i+1 to n-1

Hence $(p_{j+1}^*-h_i).q(p_{j+1}^*) =(p_{j+1}^*-h_j+h_j-h_i).q(p_{j+1}^*)= (p_j^*-h_j)q(p_j^*) +(h_j-h_i)q(p_{j+1}^*)$

$< (p_j^*-h_j)q(p_j^*) +(h_j-h_i)q(p_j^*)$ (given than $h_j>h_i$ and that $q(p_i^*)$ decreases in i) .

Hence $(p_{j+1}^*-h_i).q(p_{j+1}^*)<(p_j^*-h_i)q(p_j^*)$ for any j from i+1 to n-1 and therefore:

$(p_j^*-h_i).q(p_j^*)<(p_{i+1}^*-h_i)q(p_{i+1}^*) = (p_i^*-h_i)q(p_i^*)$ for any j from i+2 to n.

Let us now establish that t_i, for i from 2 to n-1, prefers p_i^* and p_{i+1}^* to any p_j^*, with j lower than i.

We have, for any j, with $1<j \le i$:

$$
\begin{aligned}
(p_{j-1}^*-h_i).q(p_{j-1}^*) &= (p_{j-1}^*-h_{j-1}).q(p_{j-1}^*)+(h_{j-1}-h_i)q(p_{j-1}^*)\\
&= (p_j^*-h_{j-1}).q(p_j^*)+(h_{j-1}-h_i)q(p_{j-1}^*)\\
&=(p_j^*-h_i).q(p_j^*)+ (h_i-h_{j-1})q(p_j^*)+(h_{j-1}-h_i)q(p_{j-1}^*)\\
&=(p_j^*-h_i).q(p_j^*)+ (h_i-h_{j-1})(q(p_j^*)-q(p_{j-1}^*))\\
&< (p_j^*-h_i).q(p_j^*) \text{ because } (h_j-h_{j-1})(q(p_j^*) -q(p_{j-1}^*))<0.
\end{aligned}
$$

It follows that $(p_j^*-h_i).q(p_j^*) < (p_i^*-h_i)q(p_i^*)$ for j, with $1\le j<i$.

It follows that t_i's behaviour is optimal, for i from 1 to n.

Let us now turn to the consumer. Given his out of the equilibrium path beliefs, his reaction to out of equilibrium prices is optimal. We consider now his behaviour after equilibrium prices:

It is optimal to accept H_1.

Only t_{i-1} and t_i play p_i^* for any i from 2 to n.

Accepting p_i^* leads to the expected payoff:

$\rho_{i-1}\, \pi_{i-1}(p_i^*)(H_{i-1}-p_i^*)+\rho_i\, \pi_i(p_i^*)(H_i-p_i^*)$

Given (1) this payoff is equal to 0, which justifies the buyer’s mixed strategy.

Appendix 2

Implementing $q(p_i^*)$ is straightforward and will not be reproduced here.

Let us turn to $\pi_i(p_{i+1}^*)$.

We have $\pi_n(p_n)=1$.
The buyer is indifferent between buying and refusing trade at price p_n* only if
$(H_n - p_n*)\ \rho_n + (H_{n-1}-p_n*)\ \pi_{n-1}(p_n*)\rho_{n-1}=0$, i.e.
$\pi_{n-1}(p_n*)= k/(2.(K-k/2))=b/a$ by setting $b=k/2$ and $a= K-k/2$
Hence $\pi_{n-1}(p_{n-1}*)=1-\pi_{n-1}(p_n*)= 1-b/a$
The buyer is indifferent between buying and refusing trade at price $p_{n-1}*$ only if
$(H_{n-1} - p_{n-1}*)\ \pi_{n-1}(p_{n-1}*)\rho_{n-1} + (H_{n-2}-p_{n-1}*)\ \pi_{n-2}(p_{n-1}*)\rho_{n-2}=0$, i.e.
$\pi_{n-2}(p_{n-1}*)= (1-b/a)(k/2)/(K-k/2)=(b/a) - (b/a)^2$.
Hence $\pi_{n-2}(p_{n-2}*)= 1-\pi_{n-2}(p_{n-1}*)= 1-b/a +(b/a)^2$.
A recurrence reasoning implies that :
For any i even, i from 2 to n-2 or n-1 (depending on whether n is odd or even),
$\pi_{n-i}(p_{n-i+1}*)= (b/a)-(b/a)^2 +(b/a)^3-\ldots\ldots-(b/a)^i= \delta-\delta^2 +\delta^3-\ldots\ldots-\delta^i$
$= \delta(1- (\delta^2)^{(1+(i-2)/2)})/(1-\delta^2) - \delta^2(1- (\delta^2)^{1+(i-2)/2)})/(1-\delta^2)$
$= (1- \delta^i)\delta(1- \delta)/(1-\delta^2)= \delta(1-\delta^i)/(1+\delta)$
For any odd i, i from 1 to n-1 or n-2 (depending on whether n is odd or even)
$\pi_{n-i}(p_{n-i+1}*)= (b/a)-(b/a)^2 +(b/a)^3-\ldots\ldots+(b/a)^i= \delta-\delta^2 +\delta^3-\ldots\ldots+\delta^i$
$= \delta(1- (\delta^2)^{(1+(i-1)/2)})/(1-\delta^2) - \delta^2(1- (\delta^2)^{1+(i-3)/2)})/(1-\delta^2)$
$= (\delta- \delta^{2+i}- \delta^2+\delta^{i+1})/(1-\delta^2)= (1- \delta)(\delta + \delta^{i+1})/(1-\delta^2)= \delta\ (1+\delta^i)/(1+\delta)$

Appendix 3
Let us focus on a PBE path in which each type of seller gets a positive payoff.

Let us first prove that if t_i plays 3 prices p, p' and p", then $p'=H_i$.
We necessarily have $(p-h_i)q=(p'-h_i)q'=(p''-h_i)q''$ where q, q' and q" are the probabilities of buying at prices p, p' and p". Necessarily $q >q'>q''>0$ (given the positive payoff of each type of seller). It follows that, for each type t_j with $j<i$, $(p-h_j)q>(p'-h_j)q'>(p''-h_i)q''$ and that for each type t_j with $j>i$, $(p-h_j)q<(p'-h_j)q'<(p''-h_i)q''$. Therefore p' and p" can not be played by any type lower than t_i and p and p' can not be played by any type higher than t_i. It derives that p' is only played by t_i. Given that q' is different from 0 and 1, the consumer is indifferent between buying and not buying; this is only possible if $p'=H_i$.
It follows in the same way that, if t_i plays 4 prices p, p', p" and p"', with $p<p'<p''<p'''$, then $p'=p''=H_i$. Hence each type of seller sets at most 3 prices. Moreover, if she sets three prices, the middle price is H_i.

We now show that if a price p is only played by t_i, then it is necessarily equal to H_i. As a matter of fact, if $p>H_i$, p is refused and t_i's payoff is null (a contradiction to the positivity of the payoff of each type of seller). If $p<H_i$ then p is accepted with probability 1. It follows that nobody plays a price lower than p; hence p is necessarily the lowest price played in the game. Moreover, given that t_i prefers p to any higher equilibrium price, any type lower than t_i also prefers p to the higher prices. Hence, either t_i is different from t_1 and p is played by several types (a contradiction to our assumption), either $t_i=t_1$; but the lowest price played by t_1, in each PBE, is at least H_1 (a contradiction to our assumption), given that any price lower or equal to H_1 is accepted by the consumer. It follows that if a price p is only played by t_i, then it is necessarily equal to H_i.
It derives from the above observation that if t_i plays a price p different from H_i, then p is necessarily played by another type. Let us suppose the contrary. Then, if $p<H_i$, (i is necessarily different from 1) p is accepted with probability 1 which leads all the types lower

than t_i to play p (a contradiction to our assumption). If $p>H_i$, p is refused and t_i's payoff is null, a contradiction to the positivity of the seller types' payoff. It follows that p is necessarily played by another type.
Let us be more precise by showing that an adjacent type, t_{i-1} or t_{i+1} , plays p.
If p is played by t_j with $j<i-1$, than t_{i-1} prefers p to any lower price. And, given that t_i plays p, t_{i-1} prefers p to any higher price. It follows that t_{i-1} only plays p.
Symmetrically, if p is played by a type t_j with $j>i+1$, than t_{i+1} prefers p to any higher price. And, given that t_i plays p, t_{i+1} prefers p to any lower price. It follows that t_{i+1} only plays p.

It immediately follows that at most (2n-1) different prices are played in the game. As a matter of fact, given that a type t_i can at most play 3 different prices, and given that, in this case, the middle price is necessarily H_i, t_1 can only play 2 different prices H_1 and $p_1>H_1$. Hence p_1 is necessarily played by t_2. It follows that t_2 can at most play the three prices, p_1, H_2 and $p_2>H_2$. It follows that t_3 plays p_2 and that t_3 can at most play the three messages p_2, H_3 and $p_3>H_3$. And so on, till to t_{n-1} who can at most play three prices, p_{n-2},H_{n-1} and p_{n-1}. Hence t_n plays p_{n-1} and she can at most play 2 different prices, p_{n-1} and H_n. The result follows.

Let us finally prove that in a PBE path in which each type of seller gets a positive payoff, the buyer's payoff can only be equal to 0.
It follows from the positivity of the seller types' payoff that the consumer accepts each equilibrium price with a strictly positive probability. Let us suppose that the buyer accepts an equilibrium price p* with probability 1. In that case, p* is necessarily the lowest price played in the equilibrium. Call t_i the highest type playing p*. Necessarily, $p^*\geq h_i$ and t_i plays p* with at most probability 1. Yet assumption (a) ensures that $\sum_{j=1}^{i}\frac{\rho_j H_j}{\sum_{k=1}^{i}\rho_k}< h_i\leq p^*$ for i from 2 to n. It follows that the consumer refuses p* (a contradiction), unless i is equal to 1. Yet, in that case, p* is necessarily equal to H_1 and the buyer's payoff is null . Hence each price different from H_1 is accepted with a probability lower than 1. It follows that the buyer is indifferent between buying and not buying at every equilibrium price different from H_1. This means that his payoff is equal to 0 (i.e. the payoff of the absence of trade) for any equilibrium price.

In fact the buyer's payoff is null in any PBE of the price model. Consider any price p* of the PBE equilibrium path. Either p is refused with probability 1, in which case the buyer's payoff is null. Either it is accepted with a positive probability, in which case the preceding observations ensure that the buyer's payoff is also equal to 0.

Documents de travail du BETA

2000–**01** *Hétérogénéité de travailleurs, dualisme et salaire d'efficience.*
Francesco DE PALMA, janvier 2000.

2000–**02** *An Algebraic Index Theorem for Non–smooth Economies.*
Gaël GIRAUD, janvier 2000.

2000–**03** *Wage Indexation, Central Bank Independence and the Cost of Disinflation.*
Giuseppe DIANA, janvier 2000.

2000–**04** *Une analyse cognitive du concept de « vision entrepreneuriale ».*
Frédéric CRÉPLET, Babak MEHMANPAZIR, février 2000.

2000–**05** *Common knowledge and consensus with noisy communication.*
Frédéric KŒSSLER, mars 2000.

2000–**06** *Sunspots and Incomplete Markets with Real Assets.*
Nadjette LAGUÉCIR, avril 2000.

2000–**07** *Common Knowledge and Interactive Behaviors : A Survey.*
Frédéric KŒSSLER, mai 2000.

2000–**08** *Knowledge and Expertise : Toward a Cognitive and Organisational Duality of the Firm.*
Frédéric CRÉPLET, Olivier DUPOUËT, Francis KERN, Francis MUNIER, mai 2000.

2000–**09** *Tie–breaking Rules and Informational Cascades : A Note.*
Frédéric KŒSSLER, Anthony ZIEGELMEYER, juin 2000.

2000–**10** *SPQR : the Four Approaches to Origin–Destination Matrix Estimation for Consideration by the MYSTIC Research Consortium.*
Marc GAUDRY, juillet 2000.

2000–**11** *SNUS–2.5, a Multimoment Analysis of Road Demand, Accidents and their Severity in Germany, 1968–1989.*
Ulrich BLUM, Marc GAUDRY, juillet 2000.

2000–**12** *On the Inconsistency of the Ordinary Least Squares Estimator for Spatial Autoregressive Processes.*
Théophile AZOMAHOU, Agénor LAHATTE, septembre 2000.

2000–**13** *Turning Box–Cox including Quadratic Forms in Regression.*
Marc GAUDRY, Ulrich BLUM, Tran LIEM, septembre 2000.

2000–**14** *Pour une approche dialogique du rôle de l'entrepreneur/managerdans l'évolution des PME : l'ISO comme révélateur ...*
Frédéric CRÉPLET, Blandine LANOUX, septembre 2000.

2000–**15** *Diversity of innovative strategy as a source of technological performance.*
Patrick LLERENA, Vanessa OLTRA, octobre 2000.

2000–**16** *Can we consider the policy instruments as cyclical substitutes ?*
Sylvie DUCHASSAING, Laurent GAGNOL, décembre 2000.

2001–**01** *Economic growth and CO2 emissions : a nonparametric approach.*
Théophile AZOMAHOU, Phu NGUYEN VAN, janvier 2001.

2001–**02** *Distributions supporting the first–order approach to principal–agent problems.*
Sandrine SPÆTER, février 2001.

2001–**03** *Développement durable et Rapports Nord–Sud dans un Modèle à Générations Imbriquées : interroger le futur pour éclairer le présent.*
Alban VERCHÈRE, février 2001.

2001–**04** *Modeling Behavioral Heterogeneity in Demand Theory.*
Isabelle MARET, mars 2001.

2001–**05** *Efficient estimation of spatial autoregressive models.*
Théophile AZOMAHOU, mars 2001.

2001–**06** *Un modèle de stratégie individuelle de primo–insertion professionnelle.*
Guy TCHIBOZO, mars 2001.

2001–**07** *Endogenous Fluctuations and Public Services in a Simple OLG Economy.*
Thomas SEEGMULLER, avril 2001.

2001–**08** *Behavioral Heterogeneity in Large Economies.*
Gaël GIRAUD, Isabelle MARET, avril 2001.

2001–**09** *GMM Estimation of Lattice Models Using Panel Data : Application.*
Théophile AZOMAHOU, avril 2001.

2001–**10** *Dépendance spatiale sur données de panel : application à la relation Brevets–R&D au niveau régional.*
Jalal EL OUARDIGHI, avril 2001.

2001–**11** *Impact économique régional d'un pôle universitaire : application au cas strasbourgeois.*
Laurent GAGNOL, Jean–Alain HÉRAUD, mai 2001.

2001–**12** *Diversity of innovative strategy as a source of technological performance.*
Patrick LLERENA, Vanessa OLTRA, mai 2001.

2001–**13** *La capacité d'innovation dans les regions de l'Union Européenne.*
Jalal EL OUARDIGHI, juin 2001.

2001–**14** *Persuasion Games with Higher Order Uncertainty.*
Frédéric KŒSSLER, juin 2001.

2001–**15** *Analyse empirique des fonctions de production de Bosnie–Herzégovine sur la période 1952–1989.*
Rabija SOMUN, juillet 2001.

2001–**16** *The Performance of German Firms in the Business–Related Service Sectors : a Dynamic Analysis.*
Phu NGUYEN VAN, Ulrich KAISER, François LAISNEY, juillet 2001.

2001–**17** *Why Central Bank Independence is high and Wage indexation is low.*
Giuseppe DIANA, septembre 2001.

2001–**18** *Le mélange des ethnies dans les PME camerounaises : l'émergence d'un modèle d'organisation du travail.*
Raphaël NKAKLEU, octobre 2001.

2001–**19** *Les déterminants de la GRH des PME camerounaises.*
Raphaël NK AKLEU, octobre 2001.

2001–**20** *Profils d'identité des dirigeants et stratégies de financement dans les PME camerounaises.*
Raphaël NKAKLEU, octobre 2001.

2001–**21** Concurrence Imparfaite, Variabilité du Taux de Marge et Fluctuations Endogènes.
Thomas SEEGMULLER, novembre 2001.

2001–**22** *Determinants of Environmental and Economic Performance of Firms : An Empirical Analysis of the European Paper Industry.*
Théophile AZOMAHOU, Phu NGUYEN VAN et Marcus WAGNER, novembre 2001.

2001–**23** *The policy mix in a monetary union under alternative policy institutions and asymmetries.*
Laurent GAGNOL et Moïse SIDIROPOULOS, décembre 2001.

2001–**24** *Restrictions on the Autoregressive Parameters of Share Systems with Spatial Dependence.*
Agénor LAHATTE, décembre 2001.

2002–**01** *Strategic Knowledge Sharing in Bayesian Games : A General Model.*
Frédéric KŒSSLER, janvier 2002.

2002–**02** *Strategic Knowledge Sharing in Bayesian Games : Applications.*
Frédéric KŒSSLER, janvier 2002.

2002–**03** *Partial Certifiability and Information Precision in a Cournot Game.*
Frédéric KŒSSLER, janvier 2002.

2002–**04** *Behavioral Heterogeneity in Large Economies.*
Gaël GIRAUD, Isabelle MARET, janvier 2002.
(Version remaniée du Document de Travail n°2001–**08**, avril 2001).

2002–**05** *Modeling Behavioral Heterogeneity in Demand Theory.*
Isabelle MARET, janvier 2002.
(Version remaniée du Document de Travail n°2001–**04**, mars 2001).

2002–**06** *Déforestation, croissance économique et population : une étude sur données de panel.*
Phu NGUYEN VAN, Théophile AZOMAHOU, janvier 2002.

2002–**07** *Theories of behavior in principal–agent relationships with hidden action.*
Claudia KESER, Marc WILLINGER, janvier 2002.

2002–**08** *Principe de précaution et comportements préventifs des firmes face aux risques environnementaux.*
Sandrine SPÆTER, janvier 2002.

2002–**09** *Endogenous Population and Environmental Quality.*
Phu NGUYEN VAN, janvier 2002.

2002–**10** *Dualité cognitive et organisationnelle de la firme au travers du concept de communauté.*
Frédéric CRÉPLET, Olivier DUPOUËT, Francis KERN, Francis MUNIER, février 2002.

2002–**11** *Comment évaluer l'amélioration du bien–être individuel issue d'une modification de la qualité du service d'élimination des déchets ménagers ?*
Valentine HEINTZ, février 2002.

2002–**12** *The Favorite–Longshot Bias in Sequential Parimutuel Betting with Non–Expected Utility Players.*
Frédéric KŒSSLER, Anthony ZIEGELMEYER, Marie–Hélène BROIHANNE, février 2002.

2002–**13** *La sensibilité aux conditions initiales dans les processus individuels de primo–insertion professionnelle : critère et enjeux.*
Guy TCHIBOZO, février 2002.

2002–**14** *Improving the Prevention of Environmental Risks with Convertible Bonds.*
André SCHMITT, Sandrine SPÆTER, mai 2002.

2002–**15** *L'altruisme intergénérationnel comme fondement commun de la courbe environnementale* ***à la*** *Kuznets et du développement durable.*
Alban VERCHÈRE, mai 2002.

2002–**16** *Aléa moral et politiques d'audit optimales dans le cadre de la pollution d'origine agricole de l'eau.*
Sandrine SPÆTER, Alban VERCHÈRE, juin 2002.

2002–**17** *Parimutuel Betting under Asymmetric Information.*
Frédéric KŒSSLER, Anthony ZIEGELMEYER, juin 2002.

2002–**18** *Pollution as a source of endogenous fluctuations and periodic welfare inequality in OLG economies.*
Thomas SEEGMULLER, Alban VERCHÈRE, juin 2002.

2002–**19** *La demande de grosses coupures et l'économie souterraine.*
Gilbert KŒNIG, juillet 2002.

2002–**20** *Efficiency of Nonpoint Source Pollution Instruments with Externality Among Polluters : An Experimental Study.*
François COCHARD, Marc WILLINGER, Anastasios XEPAPADEAS, juillet 2002.

2002–**21** *Taille optimale dans l'industrie du séchage du bois et avantage compétitif du bois–énergie : une modélisation microéconomique.*
Alexandre SOKIC, octobre 2002.

2002–**22** *Modelling Behavioral Heterogeneity.*
Gaël GIRAUD, Isabelle MARET, novembre 2002.

2002–**23** *Le changement organisationnel en PME : quels acteurs pour quels apprentissages ?*
Blandine LANOUX, novembre 2002.

2002–**24** *TECHNOLOGY POLICY AND COOPERATION : An analytical framework for a paradigmatic approach.*
Patrick LLERENA, Mireille MATT, novembre 2002.

2003–**01** *Peut–on parler de délégation dans les PME camerounaises ?*
Raphaël NKAKLEU, mars 2003.

2003–**02** *L'identité organisationnelle et création du capital social : la tontine d'entreprise comme facteur déclenchant dans le contexte africain.*
Raphaël NKAKLEU, avril 2003.

2003–**03** *A semiparametric analysis of determinants of protected area.*
Phu NGUYEN VAN, avril 2003.

2003–**04** *Strategic Market Games with a Finite Horizon and Incomplete Markets.*
Gaël GIRAUD et Sonia WEYERS, avril 2003.

2003–**05** *Exact Homothetic or Cobb–Douglas Behavior Through Aggregation.*
Gaël GIRAUD et John K.–H. QUAH, juin 2003.

2003–**06** *Relativité de la satisfaction dans la vie : une étude sur données de panel.*
Théophile AZOMAHOU, Phu NGUYEN VAN, Thi Kim Cuong PHAM, juin 2003.

2003–**07** *A model of the anchoring effect in dichotomous choice valuation with follow–up.*
Sandra LECHNER, Anne ROZAN, François LAISNEY, juillet 2003.

2003–**08** *Central Bank Independence, Speed of Disinflation and the Sacrifice Ratio.*
Giuseppe DIANA, Moïse SIDIROPOULOS, juillet 2003.

2003–**09** *Patents versus ex–post rewards : a new look.*
Julien PÉNIN, juillet 2003.

2003–**10** *Endogenous Spillovers under Cournot Rivalry and Co–opetitive Behaviors.*
Isabelle MARET, août 2003.

2003–**11** *Les propriétés incitatives de l'effet Saint Matthieu dans la compétition académique.*
Nicolas CARAYOL, septembre 2003.

2003–**12** *The 'probleme of problem choice' : A model of sequential knowledge production within scientific communities.*
Nicolas CARAYOL, Jean–Michel DALLE, septembre 2003.

2003–**13** *Distribution Dynamics of CO_2 Emissions.*
Phu NGUYEN VAN, décembre 2003.

2004–**01** *Utilité relative, politique publique et croissance économique.*
Thi Kim Cuong PHAM, janvier 2004.

2004–**02** *Le management des grands projets de haute technologie vu au travers de la coordination des compétences.*
Christophe BELLEVAL, janvier 2004.

2004–**03** *Pour une approche dialogique du rôle de l'entrepreneur/manager dans l'évolution des PME : l'ISO comme révélateur …*
Frédéric CRÉPLET, Blandine LANOUX, février 2004.

2004–**04** *Consistent Collusion–Proofness and Correlation in Exchange Economies.*
Gaël GIRAUD, Céline ROCHON, février 2004.

2004–**05** *Generic Efficiency and Collusion–Proofness in Exchange Economies.*
Gaël GIRAUD, Céline ROCHON, février 2004.

2004–**06** *Dualité cognitive et organisationnelle de la firme fondée sur les interactions entre les communautés épistémiques et les communautés de pratique..*
Frédéric CRÉPLET, Olivier DUPOUËT, Francis KERN, Francis MUNIER, février 2004.

2004–**07** *Les Portails d'entreprise : une réponse aux dimensions de l'entreprise « processeur de connaissances ».*
Frédéric CRÉPLET, février 2004.

2004–**08** *Cumulative Causation and Evolutionary Micro–Founded Technical Change : A Growth Model with Integrated Economies.*
Patrick LLERENA, André LORENTZ, février 2004.

2004–**09** *Les CIFRE : un outil de médiation entre les laboratoires de recherche universitaire et les entreprises.*
Rachel LÉVY, avril 2004.

2004–**10** *On Taxation Pass–Through for a Monopoly Firm.*
Rabah AMIR, Isabelle MARET, Michael TROGE, mai 2004.

2004–**11** *Wealth distribution, endogenous fiscal policy and growth : status–seeking implications.*
Thi Kim Cuong PHAM, juin 2004.

2004–**12** *Semiparametric Analysis of the Regional Convergence Process.*
Théophile AZOMAHOU, Jalal EL OUARDIGHI, Phu NGUYEN VAN, Thi Kim Cuong PHAM, Juillet 2004.

2004–**13** *Les hypothèses de rationalité de l'économie évolutionniste.*
Morad DIANI, septembre 2004.

2004–**14** *Insurance and Financial Hedging of Oil Pollution Risks.*
André SCHMITT, Sandrine SPAETER, septembre 2004.

2004–**15** *Altruisme intergénérationnel, développement durable et équité intergénérationnelle en présence d'agents hétérogènes.*
Alban VERCHÈRE, octobre 2004.

2004–**16** *Du paradoxe libéral–parétien à un concept de métaclassement des préférences.*
Herrade IGERSHEIM, novembre 2004.

2004–**17** *Why do Academic Scientists Engage in Interdisciplinary Research ?*
Nicolas CARAYOL, Thuc Uyen NGUYEN THI, décembre 2004.

2005–**01** *Les collaborations Université Entreprises dans une perspective organisationnelle et cognitive.*
Frédéric CRÉPLET, Francis KERN, Véronique SCHAEFFER, janvier 2005.

2005–**02** *The Exact Insensitivity of Market Budget Shares and the 'Balancing Effect'.*
Gaël GIRAUD, Isabelle MARET, janvier 2005.

2005–**03** *Les modèles de type Mundell–Fleming revisités.*
Gilbert KOENIG, janvier 2005.

2005–**04** *L'État et la cellule familiale sont–ils substituables dans la prise en charge du chômage en Europe ? Une comparaison basée sur le panel européen.*
Olivia ECKERT–JAFFE, Isabelle TERRAZ, mars 2005.

2005–**05** *Environment in an Overlapping Generations Economy with Endogenous Labor Supply : a Dynamic Analysis.*
Thomas SEEGMULLER, Alban VERCHÈRE, mars 2005.

2005–**06** *Is Monetary Union Necessarily Counterproductive ?*
Giuseppe DIANA, Blandine ZIMMER, mars 2005.

2005–**07** *Factors Affecting University–Industry R&D Collaboration : The importance of screening and signalling.*
Roberto FONTANA, Aldo GEUNA, Mireille MATT, avril 2005.

2005–**08** *Madison–Strasbourg, une analyse comparative de l'enseignement supérieur et de la recherche en France et aux États–Unis à travers l'exemple de deux campus.*
Laurent BUISSON, mai 2005.

2005–**09** *Coordination des négociations salariales en UEM : un rôle majeur pour la BCE.*
Blandine ZIMMER, mai 2005.

2005–**10** *Open knowledge disclosure, incomplete information and collective innovations.*
Julien PÉNIN, mai 2005.

2005–**11** *Science–Technology–Industry Links and the 'European Paradox' : Some Notes on the Dynamics of Scientific and Technological Research in Europe.*
Giovanni DOSI, Patrick LLERENA, Mauro SYLOS LABINI, juillet 2005.

2005–**12** *Hedging Strategies and the Financing of the 1992 International Oil Pollution Compensation Fund.*
André SCHMITT, Sandrine SPAETER, novembre 2005.

2005–**13** *Faire émerger la coopération internationale : une approche expérimentale comparée du bilatéralisme et du multilatéralisme*.
Stéphane BERTRAND, Kene BOUN MY, Alban VERCHÈRE, novembre 2005.

2005–**14** *Segregation in Networks.*
Giorgio FAGIOLO, Marco VALENTE, Nicolaas J. VRIEND, décembre 2005.

2006–**01** *Demand and Technology Determinants of Structural Change and Tertiarisation : An Input–Output Structural Decomposition Analysis for four OECD Countries.*
Maria SAVONA, André LORENTZ, janvier 2006.

2006–**02** *A strategic model of complex networks formation.*
Nicolas CARAYOL, Pascale ROUX, janvier 2006.

2006–**03** *Coordination failures in network formation.*
Nicolas CARAYOL, Pascale ROUX, Murat YILDIZOGLU, janvier 2006.

2006–**04** *Real Options Theory for Lawmaking.*
Marie OBIDZINSKI, Bruno DEFFAINS, août 2006.

2006–**05** *Ressources, compétences et stratégie de la firme : Une discussion de l'opposition entre la vision Porterienne et la vision fondée sur les compétences.*
Fernand AMESSE, Arman AVADIKYAN, Patrick COHENDET, janvier 2006.

2006–**06** *Knowledge Integration and Network Formation.*
Müge OZMAN, janvier 2006.

2006–**07** *Networks and Innovation : A Survey of Empirical Literature.*
Müge OZMAN, février 2006.

2006–**08** *A.K. Sen et J.E. Roemer : une même approche de la responsabilité ?*
Herrade IGERSHEIM, mars 2006.

2006–**09** *Efficiency and coordination of fiscal policy in open economies.*
Gilbert KOENIG, Irem ZEYNELOGLU, avril 2006.

2006–**10** *Partial Likelihood Estimation of a Cox Model With Random Effects : an EM Algorithm Based on Penalized Likelihood.*
Guillaume HORNY, avril 2006.

2006–**11** *Uncertainty of Law and the Legal Process.*
Giuseppe DARI–MATTIACCI, Bruno DEFFAINS, avril 2006.

2006–**12** *Customary versus Technological Advancement Tests.*
Bruno DEFFAINS, Dominique DEMOUGIN, avril 2006.

2006–**13** *Institutional Competition, Political Process and Holdup.*
Bruno DEFFAINS, Dominique DEMOUGIN, avril 2006.

2006–**14** *How does leadership support the activity of communities of practice ?*
Paul MULLER, avril 2006.

2006–**15** *Do academic laboratories correspond to scientific communities ? Evidence from a large European university.*
Rachel LÉVY, Paul MULLER, mai 2006.

2006–**16** *Knowledge flows and the geography of networks. A strategic model of small worlds formation.*
Nicolas CARAYOL, Pascale ROUX, mai 2006.

2006–**17** *A Further Look into the Demography–based GDP Forecasting Method.*
Tapas K. MISHRA, juin 2006.

2006–**18** *A regional typology of innovation capacities in new member states and candidate countries.*
Emmanuel MULLER, Arlette JAPPE, Jean–Alain HÉRAUD, Andrea ZENKER, juillet 2006.

2006–**19** *Convergence des contributions aux inégalités de richesse dans le développement des pays européens.*
Jalal EL OUARDIGHI, Rabiji SOMUN–KAPETANOVIC, septembre 2006.

2006–**20** *Channel Performance and Incentives for Retail Cost Misrepresentation.*
Rabah AMIR, Thierry LEIBER, Isabelle MARET, septembre 2006.

2006–**21** *Entrepreneurship in biotechnology : The case of four start–ups in the Upper–Rhine Biovalley.*
Antoine BURETH, Julien PÉNIN, Sandrine WOLFF, septembre 2006.

2006–**22** *Does Model Uncertainty Lead to Less Central Bank Transparency ?*
Li QIN, Elefterios SPYROMITROS, Moïse SIDIROPOULOS, octobre 2006.

2006–**23** Enveloppe *Soleau et droit de possession antérieure : Définition et analyse économique.*
Julien PÉNIN, octobre 2006.

2006–**24** *Le territoire français en tant que Système Régional d'Innovation.*
Rachel LEVY, Raymond WOESSNER, octobre 2006.

2006–**25** *Fiscal Policy in a Monetary Union Under Alternative Labour–Market Structures.*
Moïse SIDIROPOULOS, Eleftherios SPYROMITROS, octobre 2006.

2006–**26** *Robust Control and Monetary Policy Delegation.*
Giuseppe DIANA, Moïse SIDIROPOULOS, octobre 2006.

2006–**27** *A study of science–industry collaborative patterns in a large european university.*
Rachel LEVY, Pascale ROUX, Sandrine WOLFF, octobre 2006.

2006–**28** *Option chain and change management : a structural equation application.*
Thierry BURGER–HELMCHEN, octobre 2006.

2006–**29** *Prevention and Compensation of Muddy Flows : Some Economic Insights.*
Sandrine SPAETER, François COCHARD, Anne ROZAN, octobre 2006.

2006–**30** *Misreporting, Retroactive Audit and Redistribution.*
Sandrine SPAETER, Marc WILLINGER, octobre 2006.

2006–**31** *Justifying the Origin of Real Options and their Difficult Evaluation in Strategic Management.*
Thierry BURGER–HELMCHEN, octobre 2006.

2006–**32** *Job mobility in Portugal : a Bayesian study with matched worker–firm data.*
Guillaume HORNY, Rute MENDES, Gerard J. VAN DEN BERG, novembre 2006.

2006–**33** *Knowledge sourcing and firm performance in an industrializing economy : the case of Taiwan in the 1990s.*
Chia–Lin CHANG, Stéphane ROBIN, novembre 2006.

2006–**34** *Using the Asymptotically Ideal Model to estimate the impact of knowledge on labour productivity : An application to Taiwan in the 1990s.*
Chia–Lin CHANG, Stéphane ROBIN, novembre 2006.

2006–**35** *La politique budgétaire dans la nouvelle macroéconomie internationale.*
Gilbert KOENIG, Irem ZEYNELOGLU, décembre 2006.

2006–**36** *Age Dynamics and Economic Growth : Revisiting the Nexus in a Nonparametric Setting.*
Théophile AZOMAHOU, Tapas MISHRA, décembre 2006.

2007–**01** *Transparence et efficacité de la politique monétaire.*
Romain BAERISWYL, Camille CORNAND, janvier 2007.

2007–**02** *Crowding–out in Productive and Redistributive Rent–Seeking.*
Giuseppe DARI–MATTIACCI, Éric LANGLAIS, Bruno LOVAT, Francesco PARISI, janvier 2007.

2007–**03** *Co–résidence chez les parents et indemnisation des jeunes chômeurs en Europe.*
Olivia ÉKERT–JAFFÉ, Isabelle TERRAZ, janvier 2007.

2007–**04** *Labor Conflicts and Inefficiency of Relationship–Specific Investments : What is the Judge's Role ?*
Bruno DEFFAINS, Yannick GABUTHY, Eve–Angéline LAMBERT, janvier 2007.

2007–**05** *Monetary hyperinflations, speculative hyperinflations and modelling the use of money.*
Alexandre SOKIC, février 2007.

2007–**06** *Detection avoidance and deterrence : some paradoxical arithmetics.*
Éric LANGLAIS, février 2007.

2007–**07** *Network Formation and Strategic Firm Behaviour to Explore and Exploit.*
Muge OZMAN, février 2007.

2007–**08** *Effects on competitiveness and innovation activity from the integration of strategic aspects with social and environmental management.*
Marcus WAGNER, février 2007.

2007–**09** *The monetary model of hyperinflation and the adaptive expectations : limits of the association and model validity.*
Alexandre SOKIC, février 2007.

La présente liste ne comprend que les Documents de Travail publiés à partir du 1er janvier 2000. La liste complète peut être donnée sur demande.
This list contains the Working Paper writen after January 2000, 1rst. The complet list is available upon request.