Endogenous Spillovers under Cournot Rivalry and Co-opetitive Behaviors

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June 2003

ABSTRACT. – We develop a model of Cournot oligopolists with endogenous R&D spillovers where a specific type of co-opetition is introduced. The two principle factors of R§D spillovers, namely the absorptive capacity and the information-sharing parameter, are assumed to depend positively on the percentage of knowledge the firm chooses to codify and reveal. It is shown that identical firms that are rivals on the final good market do not necessarily choose the lowest level for the spillover parameters. Furthermore, there is some justification for a subsidy to knowledge codification and information-sharing. However, the latter is obtained under conditions on firms' technologies and spillover functions which ensure the emergence of symmetric solutions.

Keywords : cost reduction, endogenous spillovers, information sharing, absorptive capacity, co-opetition.

JEL Classification Numbers : D43, D62, D83, L13.

*I would like to thank Rabah Amir, Antoine Bureth, Patrick Cohendet, and one anonymous referee as well as seminar participants at BETA, University of Strasbourg, for their comments and suggestions. The usual caveat applies.

1 Introduction

In this paper we develop a model of Cournot oligopolists with endogenous R&D spillovers. Strategic cost reduction R&D with spillover effects has been studied intensively in the literature of industrial organization. Most early papers (see Kamien, M, E. Muller and I. Zang (1992), Katz (1986), d'Aspremont et Jacquemin (1988), Crépon et al. (1992), Motta (1992), Beath, Poyago-Theotoky and Ulph (1998) among others[1]) assume that firms have no control over the spillover coefficient and do not have to pay for learning from rivals. However, according to Mansfield et al. (1981) imitation costs can be substantial. Hence, the effectiveness of learning is a function of investment and a deliberate learning effort. In other words spillovers are endogenous rather than exogenous.

Spillovers are usually thought to arise through one firm's acquisition of information about the research discoveries of another firm. Two major factors bear on the extent of this spillover : the amount of information shared by the source firm of externality and the usefulness of that information for the acquiring firm. We shall refer to the first factor as the information-sharing spillover (which is the fraction of information that the firm passes to other firms) and to the second one as the firm's absorptive capacity[2], namely the fraction of the information about others' research discoveries that the firm is able to exploit.

The major contribution of this paper is to construct a model where these two spillover variables are dependent. The motivation is the following. In many cases, once a firm has undertaken R&D expenditure and made a discovery, the more it decides to share its knowledge the more it is able to absorb information about the research discoveries of other firms. To be more explicit, assume that there exists a code which allows firms to share their knowledge. As an example, consider the code to be "writing patents". By using this code the firm shares not only more information but also develops skills to read the code better : A firm is more able to read a patent written by its competitors as it becomes itself an expert in writing patents. In order to take into account this codification/information process, it seems important to study what happens when, for exogenous R&D expenditure, the two spillover parameters endogenously depend on one single strategic variable, the share of knowledge which is codified for information sharing. We refer to this as the share of codified knowledge. It is assumed to increase the information-sharing spillover but also the firm's absorptive capacity.

[1]See de Bondt (1996) for a survey.

[2]This terminology was introduced by Cohen and Levinthal (1989).

Due to this double impact of knowledge codification, this paper treats with a specific type of co-opetition, where despite firms are rivals in the final good market they choose to cooperate at some level. The notion of co-opetition[3] was expanded by Nalebuff and Brandenburger (1996). These authors develop the theme that firms may modify the rules of their games in order to achieve partial cooperation, while remaining rivals in the final good market. This notion has also been formalized by Long and Soubeyran (2001) in an asymmetric[4] oligopoly. These authors consider a two-stage game where rival firms in the final good market manipulate cooperatively their marginal costs in stage one. Here, whether firms choose cooperatively or not their share of codified knowledge in order to manipulate their marginal costs, cooperation takes place in the sense that when firms codify their knowledge they share information with others. In the sequel, we refer to the game where firms collude in stage one in order to maximize industry profit as the "pure" co-opetition game. While we refer to the game where strategic variables are chosen non cooperatively at all stages as the non-collusive game.

This paper is also related to a range of recent papers on endogenous spillovers. However in the literature the two spillover variables are independently chosen and therefore the specific type of co-opetition mentioned above is not considered. For some authors, firms can alternately control either of the two spillover variables as, for example, Katsoulacos and Ulph (1998a), Amir and Wooders (1999, 2000). Others focus on the ability of firms to control the information-sharing parameter, see, for example, De Fraja (1993), Poyago-Theotoky (1999), Kultti and Takalo (1998), Takalo (1998) and Katsoulacos and Ulph (1998b). Others focus on the ability of firms to control their absorptive capacity, see, for example, Cohen and Levinthal (1989), Kamien and Zang (2000), Kanniainen and Stenbacka (2000). More recently Jin and Troege (2002) present a model where firms choose simultaneously the two spillover variables. Nevertheless, the two variables remain independent.

The initial purpose of this paper is to show that, in contrast to this recent literature, non-collusive firms operating in the same industry do not necessarily choose the lowest level for the spillover parameters. The intuition behind this result is related to the specific type of co-opetition introduced in our model. Initially, the firm has an incentive to keep its share of codified knowledge low, since firms are rivals in the product market, an increase of the information-sharing spillover parameter makes competition fiercer by decreasing the unit costs of its competitors. However, when the firm increases

[3] This terminology is due to Ray Noorda quoted in *Electronic Business Buyer*, december 1993.

[4] Their general framework allows for both ex-ante and ex-post asymmetries.

its share of codified knowledge it also develops its skills to read the code better, hence it increases its absorptive capacity of others' research discoveries and by the way its profit. The relative strength of these two marginal effects depends on two factors.

The first one is the strength of the rivalry effect induced by a uniform decrease in unit cost across firms. Note that this effect is related to the possibility of profit shifting discussed by Seade (1985).[5] Effectively, the increase of the share of codified knowledge of one firm decreases the unit cost of all firms. Seade (1985) shows that in the case of an oligopoly with identical firms and constant unit cost, a uniform decrease in unit cost across firms will rise the industry profit only if the demand curve is not too concave. At the symmetric equilibrium, this allows to increase the profit of any firm. Hence Seade's condition ensures a low rivalry effect and a positive impact of the uniform cost reduction on the profit of any firm.

Independently of the cost of codification, this first factor is the only one to operate as long as, as assumed in most of the literature, the increase of the strategic variable of one firm generates a uniform decrease in unit cost across firms. However, in this paper, despite firms are identical the increase of the share of codified knowledge of one firm generates an asymmetric decrease in unit cost across firms. Hence, a second factor operates, namely the difference between the marginal decrease of the unit cost of the firm and the marginal decrease of the unit cost of any of its competitor.

We prove that the global effect on the profit of any firm is positive if the demand curve is not too concave (our condition being less restrictive than Seade's condition) and if the elasticity of the absorptive capacity is sufficiently high with respect to the elasticity of the information-sharing function.

The second important motivation for studying a model with endogenous spillovers is to provide a framework for examining technology policies that take the form, for example, of subsidies to information-sharing research joint ventures – a type of policy that is particularly common in the European Commission. In order to examine the rationale for such a policy we limit our study to the linear demand case.

In a first step, we consider the pure co-opetition game where in stage one firms choose cooperatively their shares of codified knowledge in order to maximize industry profit while they remain rivals in stage two. We show that, when the marginal cost of codification/information-sharing is zero[6],

[5] For a discussion of the central role of this phenomenon in co-opetiton games see also Long and Soubeyran (1997).

[6] In that case the "cost of manipulating costs" (see Long and Soubeyran, 2001) is zero.

due to the internalization of the positive R§D spillovers industry profit is maximized at any symmetric solution when firms codify the maximum level of their knowledge. This remains true as long as the marginal cost of codification/information-sharing is not too large. When this marginal cost is low, we prove also that even when the solution of the pure co-opetition game is not to codify all their knowledge, firms choose to codify more than at the non-collusive equilibrium. However, this last result crucially depends on the assumptions made on firms' technology (more specifically on the concavity of the cost reduction function and the convexity of the codification/information-sharing cost function) and on the shape of the absorptive capacity and of the information-sharing function that ensure the emergence of symmetric solutions.

Furthermore, since the reduction of the sum of firms' unit costs allows to increase the industry output and by the way the consumers' surplus, this symmetric solution with maximal spillovers parameters is socially optimal. In addition, whatever the marginal cost of codification/information-sharing, at the non collusive symmetric interior equilibrium, it is always socially optimal that firms increase their share of codified knowledge. Hence, there is some justification for a subsidy to codification and information-sharing. Again, this result is conditional on restrictions ensuring the emergence of symmetric solutions.

In the last section, we prove that the welfare benefits of a collusive behavior in stage one is reversed with distinct assumptions on firms' technology when the collusive behavior is restricted to semi-collusion. Let us recall that over the last ten years there has been a considerable interest in the role of cooperation to overcome some of the market failures associated with R&D and innovation (see, for example, Katz (1986), d'Aspremont and Jacquemin (1988), Crépon et al. (1992), Motta (1992), Beath, Poyago-Theotoky and Ulph (1998)). However, virtually all this literature treats the R&D spillovers as exogenous. For example, d'Aspremont and Jacquemin (1988) assume that the same spillover parameter applies with and without cooperation. While Motta (1992), Crépon et al. (1992), Beath, Poyago-Theotoky and Ulph (1998)) assume that when firms cooperate they achieve full information-sharing and maximal absorptive capacity. The shortcoming of this formalization is that one cannot distinguish between cooperation and maximal information-sharing/absorptive capacity. Hence, we introduce the possibility that a subset of firms chooses cooperatively their shares of codified knowledge which are not necessarily set at their highest level. Semi-collusion also takes the form of an ex-ante increase in the spillover coefficients. This type of semi-collusion has been developed by Fershtman and Gandal (1994), Friedmann and Thisse (1993), Long and Soubeyran (1998). We show that for a linear relationship between cost reduction and the spillover parameters, semi-collusion involves full information-sharing and maximal absorptive capacity. However, this result is very sensitive to the specification of the cost reduction function.

The structure of the paper is as follows : in Section 2 we set the generic model up and we characterize the non-collusive equilibrium in the most general framework. In Section 3 the non-collusive symmetric equilibrium is characterized in the linear demand case. More precisely, we specify the conditions that give the non-collusive firms the incentives to choose alternately the maximal, an intermediary and the lowest share of codified knowledge. The welfare result is established in Section 4. Finally, the possibility of semi-collusion and emergence of an asymmetric equilibrium is introduced in Section 5.

2 The model

In this section, we consider a two-stage game between n identical rival firms. In stage one, they choose their share of codified knowledge, which influences the extent of their mutual information-sharing/absorptive capacity parameters and therefore their cost structure. In the second stage, they compete in quantities. Note that, in order to focus on the incentives firms have to share information and increase their absorptive capacity, we assume that firms have already made a symmetric choice of R§D expenditure.

We consider the subgame perfect equilibria of this game. These equilibria are to be determined through backward induction. Let us solve for the equilibrium of the second stage, for any given cost structure. Note that this analysis is valid regardless of the specific formulation of the problem in stage one.

2.1 Analysis of Cournot Equilibrium in Stage Two

The inverse demand function is $P = P(Q)$ satisfying

$$P'(Q) < 0 \quad \text{and} \quad P(0) = \bar{P} > 0$$

where $Q = \sum_{j=1}^{n} q_j$ and q_j denotes firm j's output.

Let θ_j denote firm j's unit cost of production and x_j its R&D expenditure. In stage two θ_j is independent of firm j's current output but it is dependent on the level of the percentage d_k of firm k's knowledge which is codified in stage one, for $k = 1, \ldots, n$. We assume that for each j the level θ_j is restricted in the range $[\theta_{\min}, \theta_{\max}]$. The unit cost of this codification and information-sharing is $c \geq 0$. As an illustration, let us consider that the code is "writing patents". In this case, the marginal cost of codification and information-sharing is mainly the wage which is effectively constant when the firm is price-taker on the labor market. Firm j's profit is

$$\pi_j = [P(Q) - \theta_j]\, q_j - c d_j x_j.$$

Firm j knows the value of all unit costs of production θ_j with $j = 1, \dots, n$ and takes q_k with $k \neq j$ as given. It chooses q_j^* to maximize profits. The first-order condition for an interior maximum, i.e such that $q_j^* > 0$ is

$$P'(Q^*)\, q_j^* + P(Q^*) = \theta_j, \tag{1}$$

where the star indicates equilibrium values. A sufficient condition for (1) to describe a solution is : $P''(Q^*)\, q_j^* + 2P'(Q^*) \;\square\; 0$. This condition may be expressed as

$$2 - s_j \varepsilon(Q^*) \geq 0 \tag{2}$$

where $\varepsilon(Q^*) = -\frac{P''(Q^*)Q^*}{P'(Q^*)}$ is the elasticity of the slope of the demand curve and $s_j = \frac{q_j^*}{Q^*}$ is firm j's market share.

Assumption 1 *The second order condition (2) is satisfied.*

We will consider only situations where all firms produce in equilibrium and there exists a unique Cournot equilibrium. Sufficient conditions for the existence and uniqueness of Cournot equilibria are given by Kolstad and Mathiesen (1987), Gaudet and Salant (1991). Summing (1) over the n firms one obtains

$$P'(Q^*)\, Q^* + nP(Q^*) = \sum_{j=1}^{n} \theta_j \equiv \theta \tag{3}$$

where θ is the sum of the unit costs. Note that θ is restricted in the range $[n\theta_{\min}, n\theta_{\max}]$. As pointed out by Bergstrom and Varian (1985), Equation (3) shows that equilibrium industry output depends only on the sum of the unit costs, and is independent of how this sum is split between firms. To ensure uniqueness and interiority of the solution additional assumptions are imposed below.

Given m, following Long and Soubeyran (2001), we define the function

$$\psi(Q) = P'(Q)\, Q + nP(Q), \qquad \psi(0) = nP(0) > n\theta_{\max}.$$

Clearly, if ψ is a strictly decreasing function for all $Q > 0$ and if there exists some $Q^{\#} > 0$ such that $\psi(Q) < 0$ for all Q greater than $Q^{\#}$, then (3) has a unique solution $Q^* = Q^*(\theta)$ for each θ in the interval $[n\theta_{\min}, n\theta_{\max}]$. The condition that ψ is strictly decreasing can be expressed as

$$\varepsilon(Q) < n + 1 \qquad \forall Q. \tag{4}$$

This is the usual stability condition for Cournot equilibria, see, for example, Dixit (1986).

Assumption 2 *There exists a positive* $Q^{\#}$ *such that* $\psi(Q) < \theta_{\min}$ *for all* $Q > Q^{\#}$ *and for all* $Q \,\square\, Q^{\#}$*, the elasticity of the slope of the demand curve is less than* $n + 1$.

Finally, the assumption that the equilibrium output q_j^* is positive for all firms is justified if the following assumption is satisfied.

Assumption 3 *For all* θ *in the interval* $[n\theta_{\min}, n\theta_{\max}]$, $\theta_j < P(Q^*(\theta))$ *for all* j.

Note that Assumptions 1 to 3 hold, for example, for $P(Q) = Q^{-(\varepsilon-1)}$ with $1 < \varepsilon < n + 1$.

We are now ready to state a few important results. From (1) and (4), we deduce that the equilibrium output is a decreasing function of θ :

$$\frac{dQ^*}{d\theta} = \frac{1}{(n + 1 - \varepsilon(Q^*))\, P'(Q^*)} < 0,$$

as an illustration for $P(Q) = Q^{-(\varepsilon-1)}$ one has

$$Q^* = \left(\frac{n + 1 - \varepsilon}{\theta}\right)^{\frac{1}{\varepsilon-1}}$$

The equilibrium output produced by firm j is

$$q_j^* = \frac{P(Q^*) - \theta_j}{-P'(Q^*)}, \tag{5}$$

for $P(Q) = Q^{-(\varepsilon-1)}$ one gets $q_j^* = \frac{\theta(n+1-\varepsilon)^{\frac{1}{\varepsilon-1}} - \theta_j (n+1-\varepsilon)^{\frac{\varepsilon}{\varepsilon-1}}}{\theta^{\frac{\varepsilon}{\varepsilon-1}}(\varepsilon-1)}$. As a result, firm j's stage-two equilibrium profit is

$$\pi_j(q_j^*, q_{-j}^*) = -\frac{(P(Q^*) - \theta_j)^2}{P'(Q^*)} - cd_j x_j = [-P'(Q^*)]\,(q_j^*)^2 - cd_j x_j,$$

where $q_{-j} = (q_1, \ldots, q_{j-1}, q_{j+1}, \ldots, q_n)$.

2.2 Analysis of Stage One : Non-collusive Manipulations of R§D spillovers

We now describe the stage-one game.

To simplify the analysis, we focus in this section on the oligopoly case where the inverse demand function is defined by $P(Q) = Q^{-(\varepsilon-1)}$ with $1 < \varepsilon < n+1$.

We assume that for $j = 1, \ldots, n$

$$\theta_j = \bar{\theta} - r_j \,, \tag{6}$$

where r_j is the reduction in unit cost due to firm j's R&D expenditure, x_j, and the spillovers it obtains from the other firm's expenditure, x_k. The magnitude of these spillovers depends on the share of codified knowledge, d_j, of any firm j. The latter influences the spillovers in two ways ; it increases the information-sharing parameter of firm j but it also increases its absorptive capacity of others' research discoveries. Formally,

$$\begin{aligned} X_j &= x_j + \gamma(d_j) \sum_{k \neq j} \beta(d_k)\, x_k \\ r_j &= f(X_j)\,, \end{aligned} \tag{7}$$

where $\gamma\prime(d_j) > 0$, $\beta\prime(d_k) > 0, 0 < \beta(d_k) \leq 1, 0 \leq \gamma(d_j) \leq 1$ and $f'(X_j) > 0$.

Following Cohen and Levinthal (1989), we may call X_j, firm j's stock of technological and scientific knowledge[7] and $\gamma(d_j)$ firm j's absorptive capacity. We may call $\beta(d_k)$ the information-sharing function. Note that d_k can be interpreted as the percentage of its knowledge that the firm chooses to codify and reveal, while $\beta(d_k)$ is the effective percentage revealed to its competitors. The absorptive capacity, in turn, is the fraction of the knowledge revealed by its competitor that the firm is able to exploit.

Cohen and Levinthal assume that this capacity depends on the firm's R&D expenditure. Here, the firm's R&D expenditure is exogenous and it is assumed that $x_j = x$ for all j. The absorptive capacity is nevertheless endogenous and depends on the firm's share of codified knowledge. To illustrate this point let us come back to the example where the code is "writing patents". The idea is that a firm is more able to read a patent written by its competitor as it becomes itself an expert in writing patents. Hence, $\gamma(d_j)$ is an increasing function of d_j.

For what regards the information-sharing parameter, in contrast to virtually all the literature, we assume that β is endogenous and is an increasing

[7] This term is refered as firm j's effective R&D investment by Kamien et al. (1992).

function of d_k. Note that it includes the case where $\beta(d_k) = d_k$. However, this last case would correspond to a rather severe restriction, since it would mean that the firm has an absolute control on the information shared with its competitors. Typically, it holds that $\beta(d_k) > d_k$ for all $d_k \geq 0$, in particular, $\beta(0) > 0$: some knowledge transfers are beyond the control of the firm[8]. The magnitude of these involuntary spillovers depends on several factors such as the amount of knowledge incorporated in a product, the technology of the firm and, in particular, all the factors related to industrial espionage, protection of intellectual rights. Clearly, as intellectual protection is more efficient, $\beta(0)$ gets closer to 0. Note that the model also allows for a situation where $\beta(d_k) < d_k$: the amount of knowledge effectively shared by the firm is lower than the knowledge it is ready to reveal. Such a result is obtained if, for example, part of the knowledge is tacit and cannot be transcribed on a patent.

The equilibrium shares of codified knowledge depend fundamentally on the ratio of the elasticity of β with respect to the elasticity of γ and on the degree of concavity of the demand curve. These elasticities are respectively noted $\varepsilon_\beta(d)$ and $\varepsilon_\gamma(d)$. To simplify the analysis we focus on symmetric equilibria, i.e. at the equilibrium one has $d_j = d^*$ for all j.

Firm j's profit at stage-two equilibrium is

$$\pi_j\left(q_j^*, q_{-j}^*\right) = \left[-P'\left(Q^*\right)\right]\left(q_j^*\left(d_j, d_{-j}\right)\right)^2 - cd_jx = \pi_j^*\left(d_j, d_{-j}\right).$$

We first study the non-collusive equilibrium where firms choose independently their shares of codified knowledge in stage one.

At any symmetric equilibrium firms choose a maximal share of codified knowledge $d^* = 1$ if $\frac{\partial\pi_j^*(d,\ldots,d)}{\partial d_j} > 0$ for all $d \in [0,1]$, i.e.

$$\begin{aligned} &\min_{d\in[0,1]} A(d) \left[2 - \varepsilon + \frac{\left((n-1)\,\varepsilon_\gamma(d) - \varepsilon_\beta(d)\right)}{\varepsilon_\beta(d)}\left(2n - \frac{(2n-1)}{n}\varepsilon\right)\right] \\ &- \frac{cd}{\varepsilon_\beta(d)\,\beta(d)\,\gamma(d)} > 0, \end{aligned} \tag{8}$$

where $A(d) = \frac{q^*(d,\ldots,d)\,f'(X(d,\ldots,d))}{(n+1-\varepsilon)} > 0$.

Conversely, at any symmetric equilibrium firms choose a minimal share

[8] Note that when $\beta(0) = 0$, at the second-stage equilibrium, firms choose the lowest codification/information parameter.

of codified knowledge $d^* = 0$ if $\frac{\partial \pi_j^*(d,\ldots,d)}{\partial d_j} < 0$ for all $d \in [0,1]$, i.e.

$$\begin{aligned} &\min_{d\in[0,1]} A(d) \left[2 - \varepsilon + \frac{((n-1)\,\varepsilon_\gamma(d) - \varepsilon_\beta(d))}{\varepsilon_\beta(d)} \left(2n - \frac{(2n-1)}{n}\varepsilon \right) \right] \\ - \quad &\frac{cd}{\varepsilon_\beta(d)\,\beta(d)\,\gamma(d)} < 0, \end{aligned} \tag{9}$$

In order to formulate the two above conditions in terms of the fundamentals we denote $g_{\min} = \min_{d\in[0,1]} g(d)$ and $g_{\max} = \max_{d\in[0,1]} g(d)$, for any function g from $[0,1]$ into $\mathbb{R}$.

Proposition 1 *(i) If*

$$\varepsilon \ < \ \frac{2n^2}{2n-1} \quad \textit{and} \tag{10}$$

$$c \ < \ A_{\min} \left[\gamma'_{\min}\beta(0) \left(2n - \frac{(2n-1)}{n}\varepsilon \right) - \beta'_{\max}\gamma(1) \left(2 - \frac{\varepsilon}{n} \right) \right] \tag{11}$$

then there exists a unique symmetric stage-one equilibrium. At this equilibrium firms choose a maximal share of codified knowledge, i.e. $d^* = 1$.

(ii) If

$$\left\{ \begin{array}{c} \frac{2n^2}{2n-1} \leq \varepsilon < n+1 \\ \textit{or} \left\{ \begin{array}{c} \varepsilon < \frac{2n^2}{2n-1} \\ A_{\max} \left[\gamma'_{\max}\beta(1) \left(2n - \frac{(2n-1)}{n}\varepsilon \right) - \beta'_{\min}\gamma(0) \left(2 - \frac{\varepsilon}{n} \right) \right] < c \end{array} \right. \end{array} \right. , \tag{12}$$

then there exists a unique symmetric stage-one equilibrium. At this equilibrium, firms choose a minimal share of codified knowledge, i.e. $d^* = 0$.

Point (i) establishes that, in contrast to Katsoulacos and Ulph (1998a, 1998b)[9], the fact that firms may choose to maximally reveal information even in the absence of collusion does not rely on the fact that firms operate in different industry and pursue complementary research paths but on the weak

[9] As underline by Katsoulacos and Ulph (1998a, 1998b), when spillovers are endogenous it is important to distinguish between substitute and complementary research paths and between firms being located in the same industry or in different industries. These authors conclude that the only configuration where firms choose maximal spillover parameters in the absence of cooperation is when firms operate in different but complementary industries. In particular, complementary research paths do not ensure on their own maximal spillover parameters. Proposition 1 establishes that maximal spillover parameters might also emerge when non-cooperative firms operate in the same industry.

concavity of the demand curve and on a sufficiently high ratio of the elasticity of the absorptive capacity with respect to the elasticity of the information-sharing function.[10]

The intuition behind this result is related to the specific type of co-opetition introduced in our model and can be obtained by interpreting (8). Initially, the firm has an incentive to keep its share of codified knowledge low, since firms are rivals in the product market, an increase of the information-sharing spillover parameter makes competition fiercer by decreasing the unit cost of its competitors. However, when the firm increases its share of codified knowledge it also develops its skills to read the code better, hence it increases its absorptive capacity of others' research discoveries and by the way its profit. The strength of these two marginal effects depends on two factors. The first one is the value of the rivalry effect induced by a uniform decrease in unit cost across firms. The second one is the difference between the marginal decrease of the unit cost of the firm and the marginal decrease of the unit cost of any of its competitors.

To bring out the role of these two factors, let us assume first that the increase of the share of codified knowledge of firm j generates a uniform decrease in unit cost across firms, i.e. $\frac{d\theta_j}{dd_j} = \frac{d\theta_k}{dd_j}$ for all $k \neq j$. In our model, this is the case at any symmetric choice d of the firms if $(n-1)\,\varepsilon_\gamma(d) = \varepsilon_\beta(d)$, see equations (27) in the appendix. The impact on firm j's profit can then be written

$$\frac{d\pi_j}{dd_j} = \sum_{k=1}^{n} \frac{d\pi_j}{d\theta_k}\frac{d\theta_k}{dd_j} = \frac{d\theta_j}{dd_j}\sum_{k=1}^{n} \frac{d\pi_j}{d\theta_k}.$$

At any symmetric choice d of the firms one has $\frac{d\pi_j}{d\theta_k} = \frac{d\pi_k}{d\theta_j}$ for all $k \neq j$, hence

$$\frac{d\pi_j}{dd_j} = \frac{d\theta_j}{dd_j}\sum_{k=1}^{n} \frac{d\pi_k}{d\theta_j}.$$

Since $\frac{d\theta_j}{dd_j} < 0$, the impact on firm j's profit is positive if and only if $\sum_{k=1}^{n} \frac{d\pi_k}{d\theta_j} < 0$. Seade (1985) shows that in the case of an oligopoly with identical firms and constant unit cost, a uniform decrease in unit cost across firms increases the industry's profits ($\sum_{k=1}^{n} \frac{d\pi_k}{d\theta_j} < 0$) provided that the demand curve is weakly concave or convex, i.e. $\varepsilon < 2$. In fact, this condition ensures that the equilibrium price does not fall significantly. At the symmetric equilibrium, this

[10] This result is obtained when firms pursue complementary research paths. However, when firms operate in the same industry the same qualitative conclusions are obtained whether firms pursue complementary or substitute research paths, see Katsoulacos and Ulph (1998b).

allows to increase the profit of any firm. Hence, if $\varepsilon < 2$, the rivalry effect is low and the uniform cost reduction has a positive impact on the profit of any firm (the first term in between the brackets in condition (8) is positive).

Independently of the cost of codification, this effect is the only one to emerge as long as, as assumed in most of the literature (see for example Long and Soubeyran (1998)), the increase of the spillover strategic variable of one firm generates a uniform decrease in unit cost across firms. However, in this paper, despite firms are identical the increase of the share of codified knowledge of one firm generates an asymmetric decrease in unit cost across firms (as long as $(n-1)\,\varepsilon_\gamma(d) \neq \varepsilon_\beta(d)$). If $(n-1)\,\varepsilon_\gamma(d) > \varepsilon_\beta(d)$ the marginal decrease of the unit cost is stronger for the firm than for any of its competitor, hence the firm benefits more than its competitors of the cost reduction (the second term in between the brackets in condition (8) is positive).

These two marginal effects overcome the marginal cost of codification if the elasticity of the slope of the demand function is sufficiently low (condition 10), and if the absorptive capacity of the firm is relatively more sensitive than its information-sharing function to a change in the share of codified knowledge (condition (11)). Note that condition (10) is less restrictive than Seade's condition since $\frac{2n^2}{2n-1} > 2$.

Point (ii) allows to underline that a strong rivalry effect, which emerges when the demand curve is sufficiently concave, might be sufficient to give firms the incentive not to codify at all.

It is interesting to observe that if $\gamma(0) = 0$ then, as long as the marginal cost of codification and information-sharing is not too high[11], one has $\frac{\partial \pi_j^*}{\partial d_j}(0, \ldots, 0) > 0$. In this case, firms have the incentive to choose a strictly positive share of codified knowledge, i.e. $d_j^* > 0$ for all j. To be more explicit, the firm must codify and reveal this information to be able to absorb any codified and revealed knowledge of its competitors. These incentives to codify and reveal knowledge are analogous to incentives to invest in R&D studied by Cohen and Levinthal (1989) leading one firm to invest in its own R&D to be able to absorb any of the R&D output of its competitors. However, the assumption that $\gamma(0) = 0$ would be quite severe in the sense that there are R&D spillovers beyond the control of the firm.

[11] Formally, this holds as long as

$$c < \gamma'(0)\,\beta(0)\,A(0)\left(2n - \frac{2n-1}{n}\varepsilon\right)(n-1).$$

3 The Linear Demand Case

In order to outline that the set of economies fulfilling conditions (10) and (11) is non negligible we focus on economies with a linear demand function. This will also allow us to define, in this case, the conditions for the emergence of a symmetric interior stage-one equilibrium. Note that here the elasticity of the slope of the demand function is null. Hence, a uniform cost reduction induces a low rivalry effect and therefore increases the profit of any firm at the symmetric equilibrium.

The inverse demand function is written

$$P(Q) = a - bQ,$$

where a, $b \in \mathbb{R}_+$ with $a > (n+1)\,\theta^{\max} - \theta^{\min}$[12].

From the n first order conditions, we deduce the equilibrium quantities at stage-two[13]

$$q_j^* = \frac{a + \sum_{k=1}^{n} \theta_k - (n+1)\,\theta_j}{(n+1)\,b}. \tag{13}$$

The equilibrium industry output is

$$Q^* = \frac{an - \sum_j \theta_j}{(n+1)\,b}. \tag{14}$$

The firm j's equilibrium profit is

$$\begin{aligned}\pi_j^*(d_j, d_{-j}) &= b\left(q_j^*(d_j, d_{-j})\right)^2 - cd_jx \\ &= \frac{\left(a + \sum_{k=1}^{n} \theta_k(d_j, d_{-j}) - (n+1)\,\theta_j(d_j, d_{-j})\right)^2}{(n+1)^2\,b} - cd_jx\end{aligned} \tag{15}$$

At stage one, the existence of a symmetric interior equilibrium depends on the relative magnitude of the elasticity of the absorptive capacity function and of the elasticity of the information-sharing function as well as on the degree of concavity of these two functions and on the degree of concavity of the reduction cost function. This is formally stated in the subsequent proposition where stage-one equilibrium is characterized. In this proposition we make use of the following notation. For any function g, $r_g = -\frac{g''}{g'}$ is the local indicator of its degree of concavity. We assume that

$$\left\{\begin{array}{c} c_f \Box r_f(d) \\ e_{\beta\gamma} \Box \varepsilon_\gamma(d)\left[n\varepsilon_\gamma(d) - \varepsilon_\beta(d)\right] \\ c_{\beta\gamma} \Box n\varepsilon_\gamma(d)\,r_\gamma(d) - \varepsilon_\beta(d)\,r_\beta(d) \end{array}\right. ,$$

[12] This is a sufficient condition for Assumptions 1 to 3 to hold.

[13] One easily checks that this equilibrium is stable since Dixit's condition holds.

where c_f, $e_{\beta\gamma}$ and $c_{\beta\gamma} \in \mathbb{R}$. Hence, c_f is an indicator of the degree of concavity of the reduction cost function. The parameter $e_{\beta\gamma}$ indicates how relatively large is the elasticity of the capacity function and in particular how relatively large it is with respect to the elasticity of the information-sharing function. Finally, $c_{\beta\gamma}$ indicates how relatively large is the product of the elasticity and the degree of concavity of the capacity function with respect to the product of the elasticity and the degree of concavity of the information-sharing function.

Proposition 2 *(i) If,*

$$c < 2(n-1) A_{\min} \left[n\gamma'_{\min}\beta(0) - \beta'_{\max}\gamma(1)\right], \tag{16}$$

where $A_{\min} = \frac{(a-\theta_{\max}) f'_{\min}}{(n+1)^2 b}$, *then there exists a unique symmetric equilibrium. At this equilibrium firms choose to codify the maximum level of their knowledge, i.e.* $d^* = 1$.
(ii) If,

$$2(n-1) A_{\max} \left[n\gamma'_{\max}\beta(1) - \beta'_{\min}\gamma(0)\right] < c \tag{17}$$

where $A_{\max} = \frac{(a-\theta_{\min}) f'_{\max}}{(n+1)^2 b}$, *then there exists a unique symmetric equilibrium. At this equilibrium firms choose to codify the minimum level of their knowledge, i.e.* $d^* = 0$.
(iii) If

$$\begin{cases} 2(n-1) A_{\max} \left[n\gamma'(1)\beta(1) - \beta'(1)\gamma(1)\right] < c \\ c < 2A_{\min} \left[n\gamma'(0)\beta(0) - \beta'(0)\gamma(0)\right] \end{cases} \tag{18}$$

and if

$$\frac{B}{D} \square (n-1) x D c_f e_{\beta\gamma} + c_{\beta\gamma}, \tag{19}$$

where $B = \frac{x b^2 c^2 (n+1)^5}{4\left(a - \frac{\theta_{\min}}{n}\right)^3 (n-1) f'_{\min}}$ *and* $D = \min_{d\in[0,1]} \left(\frac{\beta(d)\gamma(d)}{d}\right)$, *then there exists at least one symmetric equilibrium such that* $0 < d^* < 1$. *If, in addition,*

$$\begin{cases} 0 < e_{\beta\gamma} \\ \frac{f'_{\max}}{a - \frac{\theta_{\max}}{n}} < c_f \\ (n-1) E < c_{\beta\gamma} \end{cases}, \tag{20}$$

where $E = \max_{d\in(0,1)} \frac{\varepsilon_\beta(d)\varepsilon_\gamma(d)}{d}$, *then there exists a unique symmetric equilibrium.*

Points (i) and (ii) are simply the application of Proposition 1 to the linear demand case. They show that there exists a non negligible set of economies where non-collusive firms operating in the same industry do not choose the lowest level for the spillover parameters. This is illustrated in the appendix for a linear function f and concave functions β and γ.

Point (iii) demonstrates that under some restrictions firms choose to codify and reveal an intermediary level of their knowledge. For these interior solutions to emerge at least one of the three parameters c_f, $e_{\beta\gamma}$ and $c_{\beta\gamma}$ has to be sufficiently large. This means that either the reduction cost function is sufficiently concave, either the elasticity of the capacity function is very large or significant enough with respect to the elasticity of the information-sharing function, or finally the capacity function is relatively more concave than the information-sharing function. Note, in particular, that when the three fundamental functions are linear (i.e. f', β' and γ' are constant) no interior solution can emerge due to the convexity of the firm profit function.

The proposition proves in addition that the interior solution is unique when $e_{\beta\gamma}$ is non zero and c_f and $c_{\beta\gamma}$ are sufficiently large.

4 Welfare analysis

4.1 Collusive Manipulations of R§D spillovers

We shall start the welfare analysis with the study of production efficiency. In other words, we study the pure co-opetition game, where in stage one firms choose cooperatively their shares of codified knowledge in order to maximize the industry profit while they remain rivals in stage two.

Note that equilibria of this co-opetition game might well be asymmetric. Effectively, Long and Soubeyran (1997a, 1997b, 2001) show that there is an efficiency motive for an asymmetric cost manipulation. The intuition behind this result is as follows. In a Cournot oligopoly, with a fixed number of firms each having a constant marginal cost, the equilibrium industry output in stage two depends only on the sum of their marginal costs (see Bergstorm and Varian, 1985) ; it follows that if this sum is kept constant, while some firms' marginal costs are made to increase and other firms' marginal costs are made to decrease (increasing cost dispersion), then industry output, price and total revenue will remain unchanged, and therefore industry profit (and at the same time social welfare) will rise because the same total output is now produced at lower cost (as firms with decreased marginal costs will expand their market share at the expense of firms with increased marginal costs). Here, the improvement on allocative efficiency is no longer obtained through the heterogeneity of firms' unit cost of production but rather through the internalization of the positive R§D spillovers.

The total industry profit is written

$$\pi_I^*(d_j, d_{-j}) = \sum_{j=1}^{n} \left[\frac{\left(a + \sum_{k=1}^{n} \theta_k(d_j, d_{-j}) - (n+1)\theta_j(d_j, d_{-j})\right)^2}{(n+1)^2 b} - cd_j x \right]. \tag{21}$$

When the marginal cost of codification/information-sharing is zero[14], due to the internalization of the positive R§D spillovers the industry profit is maximized at any symmetric solution when firms codify the maximum level of their knowledge. One can easily check that $\frac{\partial \pi_I^*(d,\ldots,d)}{\partial d_j} > 0$ for all $d \in [0,1]$ (see equation (30) in the appendix). This remains true as long as the marginal cost c is not too large.

Proposition 3 *(i) If*

$$c < 2A_{\min}\left(\beta(0)\gamma'_{\min} + \gamma(0)\beta'_{\min}\right), \tag{22}$$

then, there exists a unique symmetric solution maximizing industry profit, $d_j = 1$ for all j.

(ii) At any symmetric non-collusive equilibrium, with $0 < d^* < 1$, where d^* is defined by (29) in the appendix, if

$$c < 2\frac{(n+1)}{(n-1)} A_{\min}\beta'_{\min}\gamma(0) \tag{23}$$

then

$$\frac{\partial \pi_I^*(d^*, \ldots, d^*)}{\partial d_j} > 0.$$

Note that condition (22) might hold simultaneously with any of the three conditions (16), (17), or (18) and (19). This means that, under (22), industry profit is maximized when firms codify all their knowledge while they might not have chosen to do so at the symmetric non-collusive equilibrium.

Under the additional assumption that π_I^* is concave with respect to (d_j, d_{-j}), point (ii) implies that even if the solution of the pure co-opetition game is not $d_j = 1$ for all j, firms choose to codify more than at the non-collusive symmetric equilibrium. Nevertheless, it is important to underline that this

[14] In that case the "cost of manipulating costs" (see Long and Soubeyran, 2001) is zero.

result is only valid under the conditions ensuring the existence of interior symmetric solutions.

In particular, without collusion the existence of an interior symmetric equilibrium requires the conditions established in Proposition 2 (iii), namely the concavity requirements of the cost reduction function and the assumptions on the shape of the absorptive capacity and of the information-sharing function, which generate a concave profit function for any firm. Note that the profit function could still be convex if the function of codification/information cost were sufficiently concave (rather than linear as assumed in this paper).

In addition, a sufficient condition for the existence of a symmetric interior solution of the pure co-opetion game requires the concavity of π_I^* with respect to (d_j, d_{-j}) which is not ensured by the concavity of π_j^* with respect to d_j. As underlined by Amir (2000), this possible failure of joint concavity of the total payoff in the strategic variable of all firms, in spite of the concavity of each payoff in own decision, plays a crucial role in the emergence of an asymmetric solution of the co-opetition game. This was also demonstrated by Long and Soubeyran (2001). These authors consider a two-stage game where rival firms in the final good market manipulate cooperatively their marginal costs in stage one. They prove that when the industry profit is strictly convex in the strategic variable of all firms the reduction of the sum of unit costs is achieved by only one firm. When applied to our model this implies that only one firm choose to codify its knowledge at the equilibrium of the pure co-opetion game.

Hence, whether collusion in stage one induces firms to increase their share of codified knowledge crucially depends on the assumptions made on firms' technology. The possible reversion of this qualitative result with the specification of firms' technologies is demonstrated in Section 5 when collusion is restricted to semi-collusion.

4.2 Social welfare

Next, consider social welfare, defined as the sum of consumers' surplus and industry profit. Consumers' surplus is

$$\begin{aligned} S(Q^*) &\equiv \int_0^{Q^*} P(Q)\, dQ - Q^* P(Q^*) \\ &= \frac{b(Q^*)^2}{2} \end{aligned} \tag{24}$$

Since, $Q^* = Q^*(\theta(d_j, d_{-j}))$ in a Cournot equilibrium, social welfare is

$$W(d_j, d_{-j}) = \sum_{j=1}^{n} \left[\frac{\left(a + \sum_{k=1}^{n} \theta_k(d_j, d_{-j}) - (n+1)\theta_j(d_j, d_{-j})\right)^2}{(n+1)^2 b} - c d_j x \right] + \frac{b\left(Q^*(\theta(d_j, d_{-j}))\right)^2}{2} \tag{25}$$

From (25) and Proposition 3, we obtain the following result :

Proposition 4 *(i) If*

$$c < \frac{A_{\min}}{(n-1)} \left((n-1)(n(n-1)+2)\beta(0)\gamma'_{\min} + 2(2n-1)\gamma(0)\beta'_{\min} \right), \tag{26}$$

then, there exists a unique symmetric solution maximizing social welfare, $d_j = 1$ for all j.

(ii) At any symmetric non collusive equilibrium, with $0 < d^* < 1$, where d^* is defined by (29) in the appendix, one has

$$\frac{\partial W(d^*, \ldots, d^*)}{\partial d_j} > 0.$$

Note that (26) is less restrictive than (22). This is clearly obtained since the reduction of the sum of unit costs allows to increase the industry output and by the way the consumers' surplus. Hence the symmetric solution where firms codify all their knowledge is obtained for a larger set of economies when social welfare (rather than industry profit) is maximized. In addition, whatever the cost of codification/information-sharing, at the non-collusive symmetric equilibrium, it is always socially optimal that firms increase their share of codified knowledge. This result implies that the social optimum is obtained for higher shares of codified knowledge under the additional assumption that W is concave with respect to (d_j, d_{-j}).

This result gives a justification to a technology policy that takes the form of subsidies to information-sharing research joint ventures – a type of policy that is particularly common in the European Commission. This subsidy might be a reduction in social charges, a simplification of the licence process, a subsidy to the organization of industrial exhibitions or scientific conferences and so on.

It is also interesting to observe that we get the traditional result that, as the number of firms increases, the non-collusive equilibrium gets closer to the social optimum. This is obtained, since as the number of firms increases, the rivalry effect is lower, hence firms' incentive to codify their knowledge increases. Effectively, by derivating equation (29), one gets $\frac{dd^*}{dn} > 0$.

5 Collusive Manipulations of R§D spillovers within a subset of firms

We now consider a model where a subset of firms cooperate in the choice of the spillover variables. This cooperation takes the form of an increase in the spillover coefficients and a cooperative choice of the shares of codified knowledge. This formalization allows us to distinguish between cooperation and maximal information-sharing/absorptive capacity. To simplify the analysis we consider a model with only three firms. More specifically, suppose that firm 1 and firm 2 collude. Then, the stocks of technological and scientific knowledge of the two firms become

$$\begin{cases} X_1 = x + \delta\gamma(d_1)\beta(d_2)x + \gamma(d_1)\beta(d_3)x \\ X_2 = x + \delta\gamma(d_2)\beta(d_1)x + \gamma(d_2)\beta(d_3)x \end{cases},$$

where $\delta > 1$. The stock of technological and scientific knowledge of firm 3 is still given by equation (7).[15]

Again, in stage one, firms choose their share of codified knowledge and, in stage-two, they compete in quantities. Equations (1) to (5) remain valid descriptions of the equilibrium in stage two. In stage one, we assume that firm 1 and firm 2 consider firm 3's share of codified knowledge as given and collusively choose their shares of codified knowledge to maximize the sum of their stage-two Cournot equilibrium profits. This formulation is in the tradition of the theory of semi-collusion (as exemplified by the work of Friedman and Thisse, 1993, Fershtman and Gandal, 1994, Long and Soubeyran, 1998, and others).

To simplify the analysis we focus in a first step on a linear function of cost reduction, i.e. $f(X) = X$, this assumption is relaxed later on. We assume, in addition that $c = 0$. In this case, collusion gives firms the incentive to choose maximal information-sharing/absorptive capacity. This is due to the fact that collusive firms internalize the positive spillovers. Firm 3 might also increase its share of codified knowledge in order to be able to absorb its competitors' knowledge. This is shown in the following proposition.

Proposition 5 *(i) If there exists a symmetric semi-collusive equilibrium, it is necessarily* $(1,1,1)$ *; the three firms choose the maximum share of codified knowledge.*

(ii) Let us assume that $\varepsilon_{\beta}(1) \square 3\varepsilon_{\gamma}(1)$, *then* $(1,1,1)$ *is a semi-collusive equilibrium.*

[15] Hence, firms are now ex-ante asymmetric.

(iii) Let us assume that $3\varepsilon_\gamma(1) < \varepsilon_\beta(1)$. *If* $\frac{3\gamma'(0)}{\beta'(0)} \square \frac{\gamma(1)}{\beta(1)}$, *then* $(1,1,0)$ *is a semi-collusive equilibrium. If* $\frac{\gamma(1)}{\beta(1)} < \frac{3\gamma'(0)}{\beta'(0)}$ *and if* γ *is more concave than* β, *i.e.* $r_\beta(d) < r_\gamma(d)$ *for all* d, *then* $\left(1,1,d_3^{SC}\right)$ *is a semi-collusive equilibrium, where* $0 < d_3^{SC} < 1$ *and* $d^* < d_3^{SC}$ $\left(d^* > d_3^{SC}\right)$ *if* $\varepsilon_\gamma(d) < \varepsilon_\beta(d)$ $\left(\varepsilon_\gamma(d) < \varepsilon_\beta(d)\right)$ *for all* $d \in [0,1]$, d_3^{SC} *is the share of codified knowledge chosen by firm* 3, *and* d^* *denotes the share of codified knowledge at the non-collusive symmetric equilibrium.*

Let us compare these results with the ones of Proposition 2. In the case where non-collusive firms choose the maximum share of codified knowledge (condition (16) holds, more precisely one has $\varepsilon_\beta(1) \square 3\varepsilon_\gamma(1)$), the introduction of semi-collusion doesn't affect the firms' equilibrium choice. In the case where non-collusive firms choose the minimum or an intermediary level of their share of codified knowledge (points (ii) and (iii) of Proposition 2, where it holds that $3\varepsilon_\gamma(1) < \varepsilon_\beta(1)$), the introduction of semi-collusion gives firm 1 and firm 2 the incentive to choose the maximum share of codified knowledge. Whether firm 3 increases its share of codified knowledge, with the introduction of semi-collusion depends on the shape of the absorptive capacity and of the information-sharing function. Note that the emergence of an asymmetric solution is sensitive to the difference between the degree of concavity of these two functions.

However, whether semi-collusion induces the collusive firms to increase their share of codified knowledge, depend crucially on the specification of the relationship between the stock of technological and scientific knowledge and the induced reduction of production cost. For a sufficiently concave function of cost reduction, f, we prove below that the result is opposite.

Proposition 6 *(i) Consider a function* f^1, *such that* $f^{1\prime}(X) > 0$ *for all* X. *Assume that at the non-collusive equilibrium firm* 1 *and firm* 2 *choose to codify all their knowledge. Furthermore, assume that under semi-collusion they choose to reduce their share of codified knowledge. Then, this reduction of the shares of codified knowledge under semi-collusion is obtained for any function of cost reduction,* f^2, *which is a concavification of* f^1, *i.e. for any function* f^2 *such that* $f^2 = g \circ f^1$ *where* $g' > 0$ *and* $g'' < 0$.
(ii) Consider the function $f^1(X) = \ln(1+X)$. *Assume that at the non-collusive equilibrium firm* 1 *and firm* 2 *choose to codify all their knowledge. Then, at the semi-collusive equilibrium they choose to reduce their shares of codified knowledge.*

6 Conclusions

In this paper the qualitative results of the welfare analysis were conditional on assumptions ensuring the emergence of symmetric solutions, asymmetric solutions have only be considered in the case of semi-collusion in stage one with ex-ante asymmetry. One natural extension is, therefore, the study of a broader class of firms technologies in order to study the emergence of asymmetric solutions of the pure co-opetition game when firms are ex-ante identical. Effectively, Long and Soubeyran (1997a, 1997b, 2001), Amir and Wooders (1999) and Amir, Evstigneev and Wooders (2003), show that equilibria of such a co-opetition game might well be asymmetric. The intuition behind this result is as follows. In a Cournot oligopoly, with a fixed number of firms each having a constant marginal cost, the equilibrium industry output in stage two depends only on the sum of their marginal costs (see Bergstorm and Varian, 1985) ; it follows that if this sum is kept constant, while some firms' marginal costs are made to decrease (increasing cost dispersion), then industry output, price and total revenue will remain unchanged, and therefore industry profit (and at the same time social welfare) will rise because the same total output is now produced at lower cost (as firms with decreased marginal costs will expand their market share at the expense of firms with increased marginal costs). This is a second efficiency motive for cost manipulation.

In addition, even when the equilibrium of the non-collusive game is symmetric the equilibrium of the co-opetition game might well be asymmetric. This is due to the possible failure of joint concavity of the total payoff in the strategic variable of all firms, in spite of the concavity of each payoff in own decision. This phenomenon can give an account of the reversion of the qualitative results with the specification of firms' technologies obtained in a co-opetition game. Such an explanation has been offered by Amir (2000), in a two-stage Cournot oligopoly with R§D investment decisions in stage one, to the reversion of the qualitative results obtained in the co-opetition game by considering weakly decreasing returns to scale in the R§D technology rather than strongly decreasing ones. More generally, welfare results are often sensitive to the specification of firms' technologies. This dependence has been exemplified by Stahn (1998), who demonstrates that the introduction of production costs reverses the welfare benefits of standardization.

7 Appendix

7.1 Proof of Proposition 1

We shall first establish conditions (8) and (9). Let us determine the impact of a variation of the production cost on stage-two equilibrium. For all j and $k \neq j$, one deduces from (5)

$$\begin{cases} \frac{dq_j^*}{d\theta_j} = \frac{\frac{dQ^*}{d\theta}\left[P''(Q^*)(P(Q^*)-\theta_j)-(P'(Q^*))^2\right]+P'(Q^*)}{[P'(Q^*)]^2} \\ \frac{dq_j^*}{d\theta_k} = \frac{\frac{dQ^*}{d\theta}\left[P''(Q^*)(P(Q^*)-\theta_k)-(P'(Q^*))^2\right]}{[P'(Q^*)]^2} \end{cases}.$$

Thus for $Q^* = nq^*$,

$$\begin{cases} \frac{dq_j^*}{d\theta_j} = \left(n - \frac{(n-1)}{n}\varepsilon\right)\frac{dQ^*}{d\theta} \\ \frac{dq_j^*}{d\theta_k} = -\left(1 - \frac{\varepsilon}{n}\right)\frac{dQ^*}{d\theta} \end{cases}.$$

Let us now determine the impact of variations of the share of codified knowledge on production costs. From (6) and (7), we deduce that

$$\begin{cases} \frac{d\theta_j}{dd_j} = -xf'(X_j)\gamma'(d_j)\sum_{k\neq j}\beta(d_k) \\ \frac{d\theta_k}{dd_j} = -xf'(X_k)\gamma(d_k)\beta'(d_j) \end{cases}. \tag{27}$$

Hence, firm j's marginal profit is, for $d_j = d$ for all j,

$$\begin{aligned} &\frac{\partial \pi_j^*}{\partial d_j}(d,\dots,d) \\ = \; &\frac{q^* f'(X)\,x\,(n-1)}{(n+1-\varepsilon)}\left[\gamma'(d)\beta(d)\left(2n - \frac{2n-1}{n}\varepsilon\right) + \beta'(d)\gamma(d)\left(\frac{\varepsilon}{n} - 2\right)\right] \\ &-cx, \end{aligned}$$

where $X_j = X$ et $q_j^* = q^*$ for all j. On can easily check that the above expression is strictly positive for all $d \in [0,1]$ under (8) and that it is strictly negative for all $d \in [0,1]$ under (9).

Trivial computations show that (8) holds under (10) and (11), and (9) holds under (12).

7.2 Proof of Proposition 2

One can deduce from (15) with $\varepsilon = 0$ that firm j's marginal profit can be written

$$
\begin{aligned}
& \frac{\partial \pi_j^*}{\partial d_j}(dj, d_{-j}) \\
= \; & \frac{2q_j^* x}{(n+1)}\left[n f'(X_j)\gamma'(d_j)\sum_{k\neq j}\beta(d_k) - \beta'(d_j)\sum_{k\neq j} f'(X_k)\gamma(d_k)\right] - cx.
\end{aligned}
$$

If $d_j = d$ for all j, one has

$$
\frac{\partial \pi_j^*}{\partial d_j}(d,\dots,d) = \frac{2q^* x(n-1) f'(X)}{(n+1)}\left[n\gamma'(d)\beta(d) - \beta'(d)\gamma(d)\right] - cx, \tag{28}
$$

where $q_j^* = q^*$ and $X_j = X^*$ for all j.

One easily checks that $\frac{\partial \pi_j^*}{\partial d_j}(d,\dots,d) > 0$ for all $d \in [0,1]$ under condition (16) and $\frac{\partial \pi_j^*}{\partial d_j}(d,\dots,d) < 0$ for all $d \in [0,1]$ under condition (17). Points (i) and (ii) follow.

It remains to prove point (iii). Note that, in the stage-one game, the set of possible strategies of any player is $[0,1]$. Furthermore, function π_j^* is continuous in $(d_1,\dots,d_n)$. Therefore, if π_j^* is quasi-concave in d_j for any $(d_1,\dots,d_n)$ such that $d_j = d$ for all j, the traditional sufficient conditions for the existence of a symmetric Nash equilibrium hold. Note that the quasiconcavity requirement holds if for any $(d_1, d_2,\dots,d_n)$ such that $d_j = d$, $d \in [0,1]$ and $\frac{\partial \pi_j^*}{\partial d_j}(d,d,\dots,d) = 0$, one has $\frac{\partial^2 \pi_j^*}{(\partial d_j)^2}(d,d,\dots,d) \; \square \; 0$. From (28) :

$$
\begin{aligned}
& \frac{\partial^2 \pi_j^*}{(\partial d_j)^2}(d,d,\dots,d) \\
= \; & 2\frac{(n-1)^2}{b(n+1)}\left(f'(X)\right)^2 x^2\left[n\gamma'(d)\beta(d) - \gamma(d)\beta'(d)\right]^2 \\
+ \; & 2\frac{(n-1)}{(n+1)}xq^*\left[\begin{array}{c} nf''(X)\left(\beta(d)\gamma'(d)\right)^2(n-1)x + nf'(X)\gamma''(d)\beta(d) \\ -\beta(d)\gamma(d)\beta'(d)\gamma'(d)f''(X)x - \gamma(d)f'(X)\beta''(d)\end{array}\right].
\end{aligned}
$$

In addition, $\frac{\partial \pi_j^*}{\partial d_j}(d,d,\dots,d) = 0$ is obtained if and only if

$$
2q^*(d,\dots,d)f'(X(d,\dots,d))\frac{(n-1)}{(n+1)}\left(n\gamma'(d)\beta(d) - \gamma(d)\beta'(d)\right) = c. \tag{29}
$$

Thus, for any $d \in [0,1]$ with $\frac{\partial \pi_j^*}{\partial d_j}(d, d, \ldots, d) = 0$, one has

$$
\begin{aligned}
& \frac{\partial^2 \pi_j^*}{(\partial d_j)^2}(d, \ldots, d) = \frac{(cx)^2 (n+1)}{2bq^{*2}} + 2xq^* \frac{(n-1)}{(n+1)} \\
& \left[\begin{array}{c} f''(X)\, x\,(n-1)\,\beta(d)\,\gamma'(d)\,(n\beta(d)\,\gamma'(d) - \beta'(d)\,\gamma(d)) \\ + f'(X)\,(n\beta(d)\,\gamma''(d) - \beta''(d)\,\gamma(d)) \end{array} \right] \\
\Leftrightarrow \quad & \frac{\partial^2 \pi_j^*}{(\partial d_j)^2}(d, \ldots, d) = \frac{(cx)^2 (n+1)}{2bq^{*2}} + 2xq^* \frac{(n-1)}{(n+1)} \frac{f'(X)\,\beta(d)\,\gamma(d)}{d} \\
& [x(n-1)\, r_f(d)\, \varepsilon_\gamma(d)\,(n\varepsilon_\gamma(d) - \varepsilon_\beta(d)) + n\varepsilon_\gamma(d)\, r_\gamma(d) - \varepsilon_\beta(d)\, r_\beta(d)]
\end{aligned}
$$

Condition (19) ensures that $\frac{\partial^2 \pi_j^*}{(\partial d_j)^2}(d, \ldots, d) \leq 0$ for all $d \in [0,1]$ and by the way the existence of an interior symmetric equilibrium d^*, with $0 < d^* < 1$, characterized by (29). In addition, this symmetric equilibrium is unique if the right hand term of equation (29) is monotone in d. This property holds if the following expression

$$
\begin{aligned}
& \frac{2x(n-1)^2}{(n+1)^2 b} (\gamma'\beta + \gamma\beta')(n\gamma'\beta - \gamma\beta')\, f'(X)\,[f'(X) - (a - \theta_j)\, r_f(d)] \\
& + 2q^* f'(X) \frac{(n-1)}{(n+1)} (n\gamma''\beta + (n-1)\gamma'\beta' - \gamma\beta'').
\end{aligned}
$$

is of the same sign for any $d \in (0,1)$. This is true under condition (20).

8 Illustration of Proposition 2

Let us focus on the subclass of economies where the unitary cost reduction induced by the investment in R§D is constant, i.e. $f(X) = \phi X$ where $\phi > 0$, and the functions β and γ are defined by

$$
\begin{cases} \beta(d) = \beta_0 + \beta_1 d - \beta_2 d^2 \\ \gamma(d) = \gamma_0 + \gamma_1 d - \gamma_2 d^2 \end{cases},
$$

with

$$
\begin{cases} 0 < \beta_0 \leq 1 \\ 0 \leq 2\beta_2 \leq \beta_1 \\ 0 \leq \beta_0 + \beta_1 - \beta_2 \leq 1 \\ 0 \leq \gamma_0 \leq 1 \\ 0 \leq 2\gamma_2 \leq \gamma_1 \\ 0 \leq \gamma_0 + \gamma_1 - \gamma_2 \leq 1 \end{cases}.
$$

This implies, in particular, that the two functions β and γ are concave (the positive impact of codification is achieved at a decreasing rate).

In this case, condition (16) can be rewritten.

$$\begin{aligned} & \frac{c(n+1)^2 b}{2\phi(n-1)\left[a-\bar{\theta}+\phi x\left(1+(n-1)\beta_0\gamma_0\right)\right]} \\ < & \ (\beta_1 - 2\beta_0 n)\gamma_2 + (n\beta_0 - \beta_1)\gamma_1 - \beta_1\gamma_0. \end{aligned}$$

This condition holds if, for example,

$$\left\{ \begin{array}{c} \beta_1 < n\beta_0 \\ \frac{c(n+1)^2 b}{(n\beta_0-\beta_1)\left[2\phi(n-1)\left[a-\bar{\theta}+\phi x(1+(n-1)\beta_0\gamma_0)\right]\right]} + \frac{\beta_1\gamma_0+(2\beta_0 n-\beta_1)\gamma_2}{(n\beta_0-\beta_1)} < \gamma_1 \end{array} \right. .$$

One easily checks that there exists a non negligible subset of economies fulfilling these conditions. This is true, for example, for all economies in the neighborhood of an economy with $\beta_0 = \frac{1}{4}$, $\beta_1 = \frac{1}{2}$, $\beta_2 = \frac{1}{4}$, $\gamma_0 = \frac{1}{4}$, $\gamma_1 = 1$, $\gamma_2 = \frac{1}{2}$ and n sufficiently large to ensure that

$$\frac{2c(n+1)^2 b}{\phi(n-1)\left(a-\bar{\theta}+\frac{1}{8}\phi x(n+7)\right)} + \frac{5}{2} < n.$$

Using the same line of argument, one shows that there exists a non negligible subset of economies fulfilling condition (17).

8.1 Proof of Proposition 3

The impact of an infinitesimal increase of d_j on the total industry profit is

$$\frac{\partial \pi_I^*(d_j, d_{-j})}{\partial d_j} = \frac{\partial \pi_j^*(d_j, d_{-j})}{\partial d_j} + \sum_{k \neq j} \frac{\partial \pi_j^*(d_j, d_{-j})}{\partial d_k},$$

where for $d_j = d$ for all j,

$$\left\{ \begin{array}{l} \frac{\partial \pi_j^*(d,\ldots,d)}{\partial d_j} = \frac{2q^* x(n-1) f'(X^*)}{(n+1)} \left(n\gamma'(d)\beta(d) - \gamma(d)\beta'(d)\right) - cx \\ \frac{\partial \pi_j^*(d,\ldots,d)}{\partial d_k} = (n-1)\frac{2q^* x f'(X^*)}{(n+1)} \left(2\gamma(d)\beta'(d) - (n-1)\gamma'(d)\beta(d)\right) \end{array} \right. .$$

Thus

$$\frac{\partial \pi_I^*(d,\ldots,d)}{\partial d_j} = \frac{2q^* x(n-1) f'(X^*)}{(n+1)} \left(\beta(d)\gamma'(d) + \beta'(d)\gamma(d)\right) - cx. \quad (30)$$

Therefore, as long as condition (22) holds, one has $\frac{\partial \pi_I^*(d,\ldots,d)}{\partial d_j} > 0$ for all $d \in [0,1]$. This proves point (i).

For $d = d^*$, where d^* satisfies (29), one has $\frac{\partial \pi_I^*(d^*,\ldots,d^*)}{\partial d_j} > 0$ under (23), point (ii) follows.

8.2 Proof of Proposition 4

From (24) we deduce that

$$\frac{\partial S}{\partial d_j}(d,\dots,d)=\frac{nq^*xf'(X)}{(n+1)}\left((n-1)^2\gamma'(d)\beta(d)+2\gamma(d)\beta'(d)\right).$$

Hence,

$$\frac{\partial W}{\partial d_j}(d,\dots,d)=\frac{q^*xf'(X)}{(n+1)}\left[\begin{array}{c}(n-1)(n(n-1)+2)\gamma'(d)\beta(d)\\+2(2n-1)\gamma(d)\beta'(d)\end{array}\right]-cx.$$

Trivial computations allow then to deduce point (i) and point (ii).

8.3 Proof of Proposition 5

(i) One easily shows that

$$\begin{aligned}&\frac{\partial(\pi_1^*+\pi_2^*)}{\partial d_1}(d_c,d_c,d_3)\\=&\ \frac{q_c^*x}{2}\left[\begin{array}{c}f'(X_c)(2\gamma'(d_c)(\delta\beta(d_c)+\beta(d_3))+3\delta\beta'(d_c)\gamma(d_c))\\-f'(X_3)\beta'(d_c)(\delta\gamma(d_c)+2\gamma(d_3))\end{array}\right],\end{aligned}\tag{31}$$

where $q_c^*=q_1^*=q_2^*$ and $X_c=X_1=X_2$. If $f'(X)=1$, for all X, one has

$$\begin{aligned}&\frac{\partial(\pi_1^*+\pi_2^*)}{\partial d_1}(d_c,d_c,d_3)\\=&\ \frac{q_c^*x}{2}\left[2\gamma'(d_c)(\delta\beta(d_c)+\beta(d_3))+2\beta'(d_c)(\delta\gamma(d_c)-\gamma(d_3))\right].\end{aligned}\tag{32}$$

For $d_c=d_3=d$, one gets

$$\frac{\partial(\pi_1^*+\pi_2^*)}{\partial d_1}(d,d,d)=\frac{q_c^*x}{2}\left[2(\delta+1)\gamma'(d)\beta(d)+2(\delta-1)\beta'(d)\gamma(d)\right]>0,$$

for all d. This implies that at the symmetric equilibrium the two cooperative firms choose the maximum share of codified knowledge.
(ii) From point (i), it is obvious that firm 1 and firm 2 do not have any incentive to deviate from $(d_c,d_c,d_3)=(1,1,1)$. It remains to check that this is also true for firm 3 as long as $3\varepsilon_\gamma(1)>\varepsilon_\beta(1)$. One easily shows that

$$\frac{\partial\pi_3^*}{\partial d_3}(d_c,d_c,d_3)=q_3^*x\left[3f'(X_3)\gamma'(d_3)\beta(d_c)-f'(X_c)\gamma(d_c)\beta'(d_3)\right].\tag{33}$$

For $f'(X)=1$, for all X and $d_3=d_c=d$ one gets

$$\frac{\partial \pi_3^*}{\partial d_3}(d,d,d) = q_3^* x\left[3\gamma'(d)\beta(d) - \gamma(d)\beta'(d)\right].$$

Hence, $\frac{\partial \pi_3^*}{\partial d_3}(1,1,1) \geq 0$ if $3\gamma'(1)\beta(1) - \gamma(1)\beta'(1) \geq 0 \Leftrightarrow 3\varepsilon_\gamma(1) \geq \varepsilon_\beta(1)$.

(iii) Let us check that $\left(1,1,d_3^{SC}\right)$, with $0 \Box d_3^{SC} < 1$, is an equilibrium. In a first step, note that firm 1 and firm 2 don't have any incentive to deviate from $\left(1,1,d_3^{SC}\right)$. Effectively, from (32) written for $d_c=1$ and $d_3=d_3^{SC}$ one gets

$$\begin{aligned}&\frac{\partial\left(\pi_1^*+\pi_2^*\right)}{\partial d_1}\left(1,1,d_3^{SC}\right)\\ =\ &\frac{q_c^* x}{2}\left[2\gamma'(1)\left(\delta\beta(1)+\beta\left(d_3^{SC}\right)\right)+2\beta'(1)\left(\delta\gamma(1)-\gamma\left(d_3^{SC}\right)\right)\right] > 0,\end{aligned}$$

since $\delta\gamma(1) - \gamma\left(d_3^{SC}\right) > 0$ for $0 \Box d_3^{SC} \Box 1$. In a second step we study firm 3's best response to its competitors' choices $(1,1)$. If $\frac{3\gamma'(0)}{\beta'(0)} \Box \frac{\gamma(1)}{\beta(1)}$ then firm 3 does not have any incentive to deviate from $(1,1,0)$. Effectively, from (33) written for $d_c=1$ et $d_3=0$ one deduces

$$\frac{\partial \pi_3^*}{\partial d_3}(1,1,0) = q_3^* x\left[3\gamma'(0)\beta(1) - \gamma(1)\beta'(0)\right] \Box 0.$$

If $\frac{3\gamma'(0)}{\beta'(0)} > \frac{\gamma(1)}{\beta(1)}$ firm 3 might choose an intermediary share of codified knowledge d_3^{SC} with $0 < d_3^{SC} < 1$ when the two cooperative firms choose the maximum shares of codified knowledge parameters. We recall that the existence of the equilibrium $\left(1,1,d_3^{SC}\right)$ is ensured by the quasi-concavity of $\pi_3^*(1,1,d_3)$ with respect to d_3. Let us check this property. More precisely, we check that for any $d_3 \in [0,1]$ with $\frac{\partial \pi_3^*}{\partial d_3}(1,1,d_3)=0$, one has $\frac{\partial^2 \pi_3^*}{(\partial d_3)^2}(1,1,d_3) \Box 0$. For any $d_3 \in [0,1]$ with

$$\frac{\partial \pi_3^*}{\partial d_3}(1,1,d_3) = 0 \Leftrightarrow 3\gamma'(d_3)\beta(1) - \gamma(1)\beta'(d_3) = 0,$$

one has

$$\frac{\partial^2 \pi_3^*}{(\partial d_3)^2}(1,1,d_3) = q_3^* x\left[3\gamma''(d_3)\beta(1) - \gamma(1)\beta''(d_3)\right].$$

Thus, for any d_3 with $\frac{\partial \pi_3^*}{\partial d_3}(1,1,d_3)=0$, one has

$$\frac{\partial^2 \pi_3^*}{(\partial d_3)^2}(1,1,d_3) = q_3^* x\gamma(1)\left[\gamma''(d_3)\frac{\beta'(d_3)}{\gamma'(d_3)} - \beta''(d_3)\right].$$

This last expression is non positive as long as $r_\gamma(d) > r_\beta(d)$ for all d. This allows us to deduce that firm 3's choice is characterized by the first-order condition of its optimization program. However, this condition is written

$$3\frac{\gamma'\left(d_3^{SC}\right)}{\beta'\left(d_3^{SC}\right)} - \frac{\gamma(1)}{\beta(1)} = 0. \tag{34}$$

for $1 > d_3^{SC} > 0$. Furthermore, by assumption

$$\begin{cases} 3\frac{\gamma'(0)}{\beta'(0)} - \frac{\gamma(1)}{\beta(1)} > 0 \\ 3\frac{\gamma'(1)}{\beta'(1)} - \frac{\gamma(1)}{\beta(1)} < 0 \end{cases} \tag{35}$$

and the function $3\frac{\gamma'(d_3)}{\beta'(d_3)} - \frac{\gamma(1)}{\beta(1)}$ is continuous in d_3. Hence, we deduce from 35 and the theorem of intermediary values that there exists d_3^{SC}, with $1 > d_3^{SC} > 0$, solution of (34).

Note, in addition, that this solution is unique since $3\frac{\gamma'(d)}{\beta'(d)} - \frac{\gamma(1)}{\beta(1)}$ si strictly decreasing under the assumption that $r_\beta(d) < r_\gamma(d)$ for all $d \in [0,1]$. We compare now d_3^{SC} with d^*. We recall that d^* is characterized by $0 < d^* < 1$ and $\frac{\gamma'(d^*)}{\beta'(d^*)} = \frac{\gamma(d^*)}{\beta(d^*)}$. Furthermore, under the assumption that $\varepsilon_\gamma(d) < \varepsilon_\beta(d)$ for all $d \in [0,1]$, the function $\frac{\gamma(d)}{\beta(d)}$ is decreasing in d. This implies that

$$3\frac{\gamma'\left(d_3^{SC}\right)}{\beta'\left(d_3^{SC}\right)} = \frac{\gamma(1)}{\beta(1)} < \frac{\gamma(d^*)}{\beta(d^*)} = 3\frac{\gamma'(d^*)}{\beta'(d^*)}.$$

Therefore $d^* < d_3^{SC}$, since the function $\frac{\gamma'(d)}{\beta'(d)}$ is decreasing in d. Clearly, $d_3^{SC} < d^*$ if now $\varepsilon_\beta(d) < \varepsilon_\gamma(d)$ for all $d \in [0,1]$.

8.4 Proof of Proposition 6

(i) Assume that $\varepsilon_\beta(d) < 3\varepsilon_\gamma(d)$ for all d. From proposition 2, we deduce that at the symmetric equilibrium the non-cooperative firms choose $d^* = 1$. Suppose in addition that

$$\begin{aligned} & f^{1\prime}(X_c)\left(2(\delta+1)\gamma'(1)\beta(1) + 3\delta\beta'(1)\gamma(1)\right) \\ - \quad & f^{1\prime}(X_3)(\delta+2)\beta'(1)\gamma(1) < 0. \end{aligned} \tag{36}$$

From (31) and (36), we deduce that, when the function of cost reduction is f^1, firm 1 and firm 2 choose at the semi-collusive equilibrium to reduce the degree of their shares of codified knowledge. Consider now that the function of cost reduction is $f^2 = g \circ f^1$. In this case, one has

$$\begin{aligned}
&\frac{\partial\left(\pi_1^*+\pi_2^*\right)}{\partial d_1}(1,1,1) \\
= &\frac{q_c^* x}{2}\left[\begin{array}{c} g'\left(f^1\left(X_c\right)\right) f^{1\prime}\left(X_c\right)\left(2\left(\delta+1\right)\gamma'\left(1\right)\beta\left(1\right)+3\delta\beta'\left(1\right)\gamma\left(1\right)\right) \\ -g'\left(f^{1\prime}\left(X_3\right)\right) f^{1\prime}\left(X_3\right)\left(\delta+2\right)\beta'\left(1\right)\gamma\left(1\right) \end{array}\right] \quad (37)
\end{aligned}$$

We know that $X_3 < X_c$, hence $f^1(X_3) < f^1(X_c)$ and

$$g'\left(f^1\left(X_c\right)\right) < g'\left(f^1\left(X_3\right)\right). \quad (38)$$

From (36), (37) and (38), we deduce that

$$\frac{\partial\left(\pi_1^*+\pi_2^*\right)}{\partial d_1}(1,1,1) < 0.$$

Hence, for the function of cost reduction f^2, firm 1 and firm 2 choose to reduce their shares of codified knowledge under semi-collusion.

(ii) Suppose that $f^1(X) = \ln(1+X)$. Let us prove that there exist some values of the model parameters such that condition (36) holds. Note that this condition is valid if

$$\frac{\gamma'(1)}{\gamma(1)} < \frac{\beta'(1)}{\beta(1)} \frac{\left[\beta(1)\gamma(1)(\delta-1)(\delta-2)x-2(\delta-1)(1+x)\right]}{2(\delta+1)(1+x+2x\beta(1)\gamma(1))}. \quad (39)$$

By assumption we have

$$\frac{1}{3}\frac{\beta'(1)}{\beta(1)} < \frac{\gamma'(1)}{\gamma(1)}. \quad (40)$$

A necessary condition for (39) and (40) is that

$$\begin{aligned}
\frac{1}{3} &< \frac{\beta(1)\gamma(1)(\delta-1)(\delta-2)x-2(\delta-1)(1+x)}{2(\delta+1)(1+x+2x\beta(1)\gamma(1))} \\
\Leftrightarrow 4(2\delta-1)(1+x) &< \beta(1)\gamma(1)x(\delta-\delta_1)(\delta-\delta_2), \quad (41)
\end{aligned}$$

where δ_1 and δ_2 are the roots of the polynomial $3\delta^2 - 13\delta + 2$. One easily checks that $\delta_1 < 1$ and $\delta_2 > 1$. For $\delta_2 < \delta$, a necessary condition for (41) is

$$\begin{aligned}
h(\delta) &= \frac{4(2\delta-1)(1+x)}{x(\delta-\delta_1)(\delta-\delta_2)} < 1 \\
\Leftrightarrow 4(2\delta-1) &< 3x\left(\delta^2-7\delta+2\right)
\end{aligned}$$

For $\tilde{\delta}_2 < \delta$, this holds if

$$k(\delta) = \frac{4(2\delta - 1)}{3\left(\delta - \tilde{\delta}_1\right)\left(\delta - \tilde{\delta}_2\right)} < x, \tag{42}$$

where $\tilde{\delta}_1$ and $\tilde{\delta}_2$, with $\tilde{\delta}_1 < \tilde{\delta}_2$, are the roots of the polynomial $\delta^2 - 7\delta + 2$. Note that $\lim_{\delta \longrightarrow +\infty} h(\delta) = 0$ and $\lim_{\delta \longrightarrow +\infty} k(\delta) = 0$. Therefore, one can find $\delta > \tilde{\delta}_2$ sufficiently high to ensure (41) and (42). To conclude, one can δ and x such that condition (36) holds, these values must be such that

$$\left\{ \begin{array}{c} \tilde{\delta}_2 < \delta \\ \frac{4(2\delta-1)}{3\left(\delta-\tilde{\delta}_1\right)\left(\delta-\tilde{\delta}_2\right)} < x \\ \frac{4(2\delta-1)(1+x)}{(\delta-\delta_1)(\delta-\delta_2)x} < \beta(1)\gamma(1) \end{array} \right. .$$

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