The Favorite–Longshot Bias in Sequential Parimutuel Betting with Non-Expected Utility Players [1]

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Abstract

This paper analyzes a model of sequential parimutuel betting described as a two-horse race with a finite number of noise bettors and a finite number of strategic and symmetrically informed bettors. For generic objective probabilities that the favorite wins the race, a unique subgame perfect equilibrium is characterized. Additionally, two explanations for the favorite–longshot bias—according to which favorites win more often than the market's estimate of their winning chances imply—are offered. It is shown that this robust anomalous empirical regularity might be due to the presence of transaction costs and/or to strategic bettors' subjective attitude to probabilities.

Keywords: Parimutuel betting; Sequential decisions; Favorite–longshot bias; Non-expected utility under risk.

JEL Classification: C72; D81.

Résumé

Nous considérons un modèle séquentiel de pari-mutuel décrit comme une course à deux chevaux, avec un nombre fini de parieurs symétriquement informés. En présence de liquidité, nous montrons que l'unique équilibre parfait en sous-jeux est caractérisé par un biais en faveur de l'outsider dans le cas où des coûts de transaction existent et/ou lorsque les parieurs pondèrent subjectivement les probabilités.

1 Introduction

Empirical and theoretical research on racetrack betting has been expanded during the last twenty years due to the importance of the industry and, more generally, to the recent rise of gambling opportunities around the world. Horserace betting markets also capture important elements of investment decisions under uncertainty and they possess several usual attributes of financial markets. For example, they are characterized by a large number of investors (bettors) acting in a rich interactive and uncertain environment. Another interesting feature of racetrack betting markets is that prices (odds) of a particular horse are a decreasing function of the total amount bet on that horse. This means that rational participants consider the negative effect of their bets on their expected earnings. As they take into account the impact of their actions on odds, these participants can be assimilated to strategic traders in the recent literature on market microstructure where the process of price formation is explicitly modeled (Kyle, 1985; Glosten and Milgrom, 1985). More indirectly, the study of racetrack betting may help to understand traders' behaviors in financial markets because the increased frequency of actual gambling may have potentially important effects on changed attitudes toward risk taking in stock market investments.

Contrary to stock markets, racetrack betting markets are conveniently characterized by a well-defined end-point at which each bet possesses a definite value. More generally, horserace betting markets provide a paradigmatic example of a case where the organization of the market determines the game form and the type of competition. Hence, since the "rules of the game" driving horserace betting markets are unambiguously defined, such markets provide a useful perspective for theoretical and empirical economic analyses.

Several empirical studies have provided evidence that most racetrack betting markets do not satisfy weak form efficiency because favorites win more often than the market's estimate of their winning chances imply. This observation implies that the expected return per unit of money bet on a horse increases with the probability of the horse winning. In other words, higher average returns could be earned by betting on favorites (generally identified by lower odds) than by betting on horses with a lower probability to win (generally identified by higher odds). Such a phenomenon is known in the literature as the *favorite–longshot bias*. Among the reasons provided in the literature to explain why favorites win more often than the betting odds indicate, one can find arguments that turn on risk-loving preferences, context specific behaviour, overconfidence, extra utility from betting on longshots, bettors' tendency to discount a fix fraction of their losses, optimal responses by bookmakers (insider trading), etc.[1]

This paper proposes two theoretical explanations for the favorite–longshot bias in a model of sequential parimutuel betting. We first show that this robust anomalous empirical regularity might be due to the presence of *transaction costs*. Such an explanation was already proposed by Hurley and McDonough (1995, 1996). However, by testing experimentally the implications of their theoretical modelling, the latter authors rejected this argument. Alternatively, we show that the bias might result from bettors' *subjective attitude to probabilities*. Indeed, numerous empirical studies have provided evidence that biases in subjective odds result from the fact that bettors are oversensitive to the chances of winning on longshots and oversensitive to the chances of losing on favorites.[2] By building a game-theoretical framework where non-expected utility players interact, we show that this simple argument is strongly appealing.

Our model retains the basic features of the parimutuel system considered by Watanabe, Nonoyama, and Mori (1994). In particular, each bettor can choose between betting on one of two horses or withdrawing from betting. However, since we consider a common prior belief on the winning chances of each horse, noises bettors are introduced in order to avoid the unique no-betting equilibrium obtained when all bettors have consistent beliefs and are perfectly rational. Another distinction with Watanabe et al.'s (1994) theoretical framework is that in our model bets are placed sequentially rather than simultaneously. This feature, which has been introduced by Feeney and King (2001) in a game where players cannot refrain from betting, captures more realistically the working of racetrack betting markets where odds are listed on a tote board which is updated about once a minute. Besides, sequential choices allow the characterization of a unique (subgame perfect) equilibrium. It is also worth noticing that, contrary to most theoretical works analyzing parimutuel systems, we consider a finite number of bettors, which implies that each of them

[1] For more details on these possible explanations, see, e.g., Williams (1999).

[2] See, for example, Ali (1977), Thaler and Ziemba (1988), and Jullien and Salanié (2000).

cannot ignore the effect of his betting choice on odds.[3] Finally—so far as we know, for the first time—we allow players to subjectively weight winning chances of both horses.

The paper is organized as follows. In Section 2 we present the sequential betting model. In Section 3 we introduce bettors' subjective attitudes to probability, we characterize the equilibrium of the market accordingly, and we discuss the effects of transaction costs and probability distortions on equilibrium subjective probabilities. Concluding remarks are given in Section 4.

2 A Model of Sequential Betting

We consider a horse race between two horses called F (the favorite) and L (the longshot), with respective *objective probabilities* of winning the race p and $1-p$, where $p > 1/2$.

We consider two classes of bettors. First, there is a finite set of *strategic bettors* (or simply bettors), $N = \{1, \ldots, n\}$, who place their bets sequentially, at a predefined date. A strategic bettor maximizes his decision-weighted gain, i.e., he maximizes a modified mathematical expectation of his gain where objective probabilities are replaced by subjective weights. Second, there is a finite set of *noise bettors*, $\{1, 2, \ldots, 2K\}$, who act for exogenous motives and without regard for expected gains.[4] For convenience, we assume that noise bettors split their bets up equally amongst the two horses, i.e., K noise bettors bet one unit of money on horse F and K noise bettors bet one unit of money on horse L.

Each (strategic) bettor has the option to bet one unit of money on the favorite or to bet one unit of money on the longshot or to refrain from betting. More precisely, each bettor $i \in N = \{1, \ldots, n\}$ chooses an action $s_i \in S_i = \{F, L, D\}$ in period i, where F stands for "betting one unit of money on the favorite", L for "betting one unit of money on the longshot", and D for "withdrawing". Denote by $s^k = \{s_1, \ldots, s_k\}$ the vector of actions chosen by the first k bettors (i.e., the history of length k), and write $s^0 = \emptyset$ and $s^n = s$. Let $S^k = \prod_{i=1}^{k} S_i$ be the set of histories of length k, and write $S^0 = \{\emptyset\}$ and $S^n = S$. When a bettor $i \in N$ acts, he observes

[3] Simultaneous parimutuel betting with a continuum of bettors has been analyzed by Watanabe (1997).

[4] As shown latter, without the presence of noise bettors the market breaks down. The noise trader approach is discussed and justified in Shleifer and Summers (1990).

a history $s^{i-1} \in S^{i-1}$ of bets made by bettors $1, \ldots, i-1$.

For any history $s^k \in S^k$ of length $k \in N$, we partition $\{1, \ldots, k\}$ into three sets as

$$\begin{aligned} F(s^k) &= \{i \in \{1, \ldots, k\} : s_i = F\}, \\ L(s^k) &= \{i \in \{1, \ldots, k\} : s_i = L\}, \\ D(s^k) &= \{i \in \{1, \ldots, k\} : s_i = D\}. \end{aligned}$$

Let $F(\emptyset) = L(\emptyset) = D(\emptyset) = \emptyset$. Hence, in period k, $F(s^k)$ (respectively $L(s^k)$) is the set of bettors who have bet on horse F (respectively horse L), and $D(s^k)$ is the set of bettors who did not bet. For any history s^k, let $n_F(s^k) = |F(s^k)|$ (respectively $n_L(s^k) = |L(s^k)|$) denote the number of bettors who have bet on horse F (respectively horse L), and let $n_D(s^k) = |D(s^k)|$ denote the number of bettors who have withdrawn. Of course, $n_F(s^k) + n_L(s^k) + n_D(s^k) = k$ for all $s^k \in S^k$, $k \in N$.

In the parimutuel wagering system used at most racetracks throughout the world, bettors bet against one another and, according to the principle of mutualization, the winners share the stake money, after deductions have been made for the market maker. We denote by $t \in [0, 1[$ this level of transaction costs, i.e., the amount that the racetrack subtracts from each unit of money bet for expenses, taxes, and profit. Hence, given the sequence of bets of strategic bettors, $s \in S$, and the number of noise bettors, $2K$, the gross return to a winning one unit of money bet on horse $h \in \{F, L\}$ is given by

$$R_h(s) = (1-t)\frac{n_F(s) + n_L(s) + 2K}{n_h(s) + K}.$$

Let $O_h(s) = R_h(s) - 1$ denote the *final odds* against horse $h \in \{F, L\}$, which measure the net return per unit of money wagered.[5] Subjective probabilities refer to the betting market's estimate of each horse's chance of winning the race.[6] We

[5] Since the sum of prices implied in the odds, $\sum_h 1/(O_h + 1)$, is greater than one, the average bettor trades at a loss, which implies that the track secures a profit overall. For this reason, without the presence of noise bettors, the market necessarily breaks down since everybody would drop from betting.

[6] This term has its root in the constant returns model, where it is assumed that the transaction costs are equal to zero, and that all agents are fully informed, have identical risk-neutral preferences,

denote by

$$P_h(s) = \frac{n_h(s) + K}{n_F(s) + n_L(s) + 2K},$$

the *subjective probability* of horse $h \in \{F, L\}$. The concept of subjective probability of a horse has been widely used in the racetrack betting literature. In particular, when the favorite–longshot bias is observed, the subjective probability of the favorite is lower than its objective probability, and the subjective probability of the longshot is higher than its objective probability.

3 Characterizations of Equilibria

In this section we define bettors' strategies and their decision-weighted utilities. Then, the unique subgame perfect equilibrium is characterized according to objective probabilities, the number of noise bettors, the level of transaction costs, and strategic bettors' tendency to subjectively weight objective probabilities.

Bettors i's *behavioral strategy*, for $i \in N$, is denoted $\sigma_i : S^{i-1} \to S_i$, and a profile of behavioral strategies is denoted by $\sigma = (\sigma_1, \ldots, \sigma_n)$. Let $s(\sigma \mid s^k)$ be the final history (outcome) reached according to the profile of behavioral strategies σ, given the history $s^k \in S^k$, and let $s(\sigma \mid \emptyset) = s(\sigma)$ be the final history generated according to σ.[7]

We assume that (strategic) bettors convert objective probabilities into subjective decision weights. The decision weight attached to each state, either horse F or horse L wins the race, is determined by a *probability weighting function* $\pi : [0,1] \to [0,1]$ which transforms the individual probabilities of each consequence into weights. We further assume the following (inverted) S-shaped decision-weighting function:

$$\pi(p) = \frac{p^\gamma}{p^\gamma + (1-p)^\gamma}, \tag{1}$$

where $\gamma \in \,]0,1]$.[8] We are drawn to this partly because empirical research on individual decision making over a period of fifty years, from Preston and Baratta (1948) to Gonzalez and Wu (1999), lends support to such an inverse-S-shaped form

and maximize wealth. See Sauer (1998) for more details.

[7]Note that it is equivalent for each bettor to observe either all the decisions taken by previous bettors in the sequence or only the odds at each previous date.

[8]This form has been suggested by Quiggin (1982).

in non-expected utility models,[9] and partly because of convenience. Such a weighting function exhibits greater sensitivity to high and low probabilities relative to mid-range probabilities, and is concave below one-half and convex above it. This distortion is increasing with the difference $1-\gamma$ and implies that bettors overweight small probabilities and underweight high ones.

Accordingly, given the sequence of bets $s \in S$, bettor i's *decision-weighted utility* (or simply utility) is given by

$$V_i(s) = \begin{cases} \pi(p)O_F(s) - \pi(1-p), & \text{if } s_i = F \\ \pi(1-p)O_L(s) - \pi(p), & \text{if } s_i = L \\ 0, & \text{if } s_i = D, \end{cases}$$

which reduces to

$$V_i(s) = \begin{cases} \pi(p)(1+O_F(s)) - 1, & \text{if } s_i = F \\ \pi(1-p)(1+O_L(s)) - 1, & \text{if } s_i = L \\ 0, & \text{if } s_i = D. \end{cases}$$

With the aforementioned class of probability weighting functions, bettor i's decision-weighted utility is given by

$$V_i(s) = \begin{cases} \dfrac{(1-t)p^\gamma}{p^\gamma + (1-p)^\gamma} \dfrac{n_F(s) + n_L(s) + 2K}{n_F(s) + K} - 1, & \text{if } s_i = F \\ \dfrac{(1-t)(1-p)^\gamma}{p^\gamma + (1-p)^\gamma} \dfrac{n_F(s) + n_L(s) + 2K}{n_L(s) + K} - 1, & \text{if } s_i = L \\ 0, & \text{if } s_i = D. \end{cases}$$

Note that because (strategic) bettors face only two possible states, they behave accordingly to rank-dependent expected utility maximizers.[10] Hence, the bettors' behavior doesn't lead to violations of stochastic dominance. Of course, when $\gamma = 1$, bettors are simply expected utility maximizers.

A *subgame perfect equilibrium* (or simply equilibrium) is a profile of behavioral strategies σ such that for all $i \in N$, $s_i \in S_i$, and $s^{i-1} \in S^{i-1}$ we have

[9] See Starmer (2000) for a survey of non-expected utility theories under risk.

[10] Axiomatizations of rank-dependent expected utility have been presented, among others, by Segal (1990), Wakker (1994), and Abdellaoui (2001).

$$V_i\left(s\left(\sigma \mid s^{i-1}, \sigma_i(s^{i-1})\right)\right) \geq V_i\left(s\left(\sigma \mid s^{i-1}, s_i\right)\right).$$

To simplify the exposition, we assume as a tie-breaking rule that a bettor who expects zero utility from betting chooses to withdraw. This assumption is made without loss of generality as long as generic objective probabilities are considered.

The next lemma shows that strategic bettors never bet on both horses. This implies that the clustering of behavior obtained in the sequential parimutuel game of Feeney and King (2001), where a first group of bettors bet on one horse and subsequent bettors bet on the other horse, breaks down whenever bettors are allowed to refrain from betting. This result also contrasts with Watanabe et al. (1994) where both types of betting choices are possible at equilibrium because bettors hold mutually inconsistent beliefs.

Lemma 1 *There is no equilibrium outcomes in which some strategic bettors bet on the favorite and some strategic bettors bet on the longshot.*

Proof. Assume by way of contradiction that s, where $s_i = F$ and $s_j = L$ for some i, $j \in N$, is an equilibrium outcome. This implies that

$$V_i(s) > 0, \quad \text{for all } i \in F(s) \cup L(s).$$

Since $\pi(1-p) = 1 - \pi(p)$, we get

$$\begin{aligned}(1-t)\pi(p)\frac{n_F(s)+n_L(s)+2K}{n_F(s)+K} - 1 &> 0,\\ (1-t)(1-\pi(p))\frac{n_F(s)+n_L(s)+2K}{n_L(s)+K} - 1 &> 0,\end{aligned}$$

or, equivalently,

$$\begin{aligned}\pi(p) &> \frac{n_F(s)+K}{(n_F(s)+n_L(s)+2K)(1-t)},\\ \pi(p) &< \frac{n_F(s)+K-t(n_F(s)+n_L(s)+2K)}{(n_F(s)+n_L(s)+2K)(1-t)},\end{aligned}$$

which is impossible for all admissible parameters. □

The next lemma shows that strategic bettors never bet on the longshot if none of them bet on the favorite.

Lemma 2 *There is no equilibrium outcomes in which some strategic bettors bet on the longshot and no strategic bettors bet on the favorite.*

Proof. Let $n_F(s) = 0$. A player who bets on the longshot has strictly positive utility if and only if

$$\pi(p) < \frac{K - t(n_L(s) + 2K)}{(n_L(s) + 2K)(1 - t)},$$

which is impossible since $\frac{K-t(n_L(s)+2K)}{(n_L(s)+2K)(1-t)} \leq 1/2$ and $\pi(p) > 1/2$. $\square$

From the two previous lemmas we know that strategic bettors never bet on the longshot. Hence, we get the following result.

Lemma 3 *There is no equilibrium outcomes in which some strategic bettors bet on the longshot.*

Proof. Directly from Lemmas 1 and 2. $\square$

Thus, the equilibrium outcome is necessarily characterized by bets on the favorite or by withdrawals. Of course, strategic bets on the favorite are only observed if the number of noise bettors is sufficiently large relatively to the level of transaction costs. The next proposition gives a necessarily condition for the existence of an equilibrium characterized by bets on the favorite. In particular, it is shown that if there is no noise bettors, then the only equilibrium is for all strategic bettors to drop from betting.

Proposition 1 *If s is an equilibrium outcome and at least one strategic bettor chooses to bet on the favorite (i.e., $n_F(s) \geq 1$), then $t \leq \frac{K}{2K+1}$.*

Proof. First note that if s is an equilibrium outcome, then $n_L(s) = 0$ from Lemma 3. Assume by way of contradiction that $t > \frac{K}{2K+1}$ and $n_F(s) \geq 1$. The first inequality gives

$$
\begin{aligned}
& \frac{K+1}{(2K+1)(1-t)} > 1 & \\
\Longrightarrow\quad & \pi(p) < \frac{K+1}{(2K+1)(1-t)}, & \text{since } \pi(p) \le 1 \\
\Longrightarrow\quad & \pi(p) < \frac{K+n_F(s)}{(2K+n_F(s)+n_L(s))(1-t)}, & \text{since } n_F(s) \ge 1 \text{ and } n_L(s) = 0 \\
\Longrightarrow\quad & V_i(s) < 0, & \text{for all } i \in F(s).
\end{aligned}
$$

Hence, each bettor $i \in F(s)$ deviates by withdrawing, a contradiction with the fact that s is an equilibrium outcome. □

The next theorem gives a complete characterization of the equilibrium outcome. The equilibrium pattern of behavior is relatively simple. When the objective probability of the favorite reaches high probability intervals, then the number of bets on the favorite increases. Since bettors who bet on the favorite obtain a strictly positive utility and others get zero utility, it is intuitively clear that the equilibrium exhibits a first mover advantage, the first bettors in the sequence choosing to bet on the favorite until its odd is too low to expect positive utility. The proof of this result needs some additional lemmas; the complete arguments and calculations are given in the appendix.

Theorem 1 *Let σ be a subgame perfect equilibrium and let $k \in \{1, \dots, n-1\}$.*

1. *If $p < \frac{(K+1)^{1/\gamma}}{(K-t(2K+1))^{1/\gamma}+(K+1)^{1/\gamma}}$, then $s(\sigma) = (D, \dots, D)$.*
2. *If $p \in \left]\frac{(K+k)^{1/\gamma}}{(K-t(2K+k))^{1/\gamma}+(K+k)^{1/\gamma}}, \frac{(K+k+1)^{1/\gamma}}{(K-t(2K+k+1))^{1/\gamma}+(K+k+1)^{1/\gamma}}\right[$, then $s(\sigma) = (F, \dots, F, D, \dots, D)$, where $n_F(s^k) = k$.*
3. *If $p > \frac{(K+n)^{1/\gamma}}{(K-t(2K+n))^{1/\gamma}+(K+n)^{1/\gamma}}$, then $s(\sigma) = (F, \dots, F)$.*

Proof. See the appendix. □

A first obvious consequence of Theorem 1 is that the subjective probability of a horse is increasing with its objective probability. We also remark that if there is no transaction costs ($t = 0$) and if strategic bettors maximize their expected gains ($\gamma = 1$), then the equilibrium subjective probability of each horse is close to its objective probability. Indeed, in that case, if $p \in \left]\frac{K+k}{2K+k}, \frac{K+k+1}{2K+k+1}\right[$, then the subjective probability of the favorite is $P_F(s) = \frac{K+k}{2K+k}$. The same argument applies for objective and subjective probabilities of the longshot. The following examples illustrate this result with two and three strategic bettors and with $K = 2$ and $K = 4$.

Example 1 Let $n = K = 2$, $\gamma = 1$, and $t = 0$. Depending on the value of the favorite's objective probability, p, the equilibrium outcomes and subjective probabilities of the favorite are given by Figure 1 on page 18.

Example 2 Let $n = 3$, $K = 4$, $\gamma = 1$, and $t = 0$. Depending on the value of the favorite's objective probability, p, the equilibrium outcomes and subjective probabilities of the favorite are given by Figure 2 on page 18.

Assume now that transaction costs are strictly positive, i.e., $t > 0$, and assume again that strategic bettors maximize their expected gains, i.e., $\gamma = 1$. From Theorem 1, if $p \in \left]\frac{K+k}{(2K+k)(1-t)}, \frac{K+k+1}{(2K+k+1)(1-t)}\right[$, then the subjective probability of the favorite is $P_F(s) = \frac{K+k}{2K+k}$, which becomes smaller than p as t increases. Therefore, in this model, when transaction costs increase, the favorite–longshot bias appears at equilibrium. The next example illustrates this point with the same parameters as Example 2, but with $t = 1/4$, which is approximately the level of transaction costs of racetrack betting markets in France.

Example 3 Let $n = 3$, $K = 4$, $\gamma = 1$, and $t = 1/4$. Depending on the value of the favorite's objective probability, p, the equilibrium outcomes and subjective probabilities of the favorite are given by Figure 3 on page 18. A comparison of Figures 2 and 3 shows the tendency of strategic bettors to refrain from betting when there are significant transaction costs, resulting in a decrease of the favorite's subjective probability.

Finally, consider that there is no transaction costs but that strategic bettors subjectively weight the prior objective probabilities according to the (inverted) S-shaped decision-weighting function π given by Equation (1). From Theorem 1, if $p \in \left]\frac{(K+k)^{1/\gamma}}{K^{1/\gamma}+(K+k)^{1/\gamma}}, \frac{(K+k+1)^{1/\gamma}}{K^{1/\gamma}+(K+k+1)^{1/\gamma}}\right[$, then the subjective probability of the favorite is $P_F(s) = \frac{K+k}{2K+k}$, which becomes smaller than p as γ decreases. Hence, the more the bettors "distort" true winning chances of both horses the larger the favorite–longshot bias. The next example illustrates this bias with the same parameters as Example 2, but with $\gamma = 1/2$.

Example 4 Let $n = 3$, $K = 4$, $\gamma = 1/2$, and $t = 0$. Depending on the value of the favorite's objective probability, p, the equilibrium outcomes and subjective probabilities of the favorite are given by Figure 4 on page 18. A comparison of Figures 2 and 4 shows that probability distortions generate the favorite–longshot bias because strategic bettors tend to refrain from betting even with relatively high objective probabilities of the favorite.

More generally, the effects of transaction costs or (and) probability distortions are summarized in the following proposition.

Proposition 2 *The difference between the objective probability and the subjective probability of the favorite (longshot) is increasing (decreasing) with transaction costs, t, and bettors' probability distortions, $1 - \gamma$.*

Proof. Directly from Theorem 1 □

4 Concluding Remarks

In this paper we have analyzed a simple model of sequential parimutuel betting in which non-expected utility players either bet on one of two horses or withdraw. To avoid the no-betting equilibrium, noise bettors have been introduced. We have shown that the favorite–longshot bias (according to which favorites are underbet and longshots overbet) may be observed at the (unique) equilibrium due to the presence of transaction costs and/or to bettors' tendency to subjectively weight horses' winning chances. While there is some empirical evidence suggesting that the favorite–longshot bias is not explained by transaction costs, the probability distortion argument still remains appealing. Indeed, such an explanation is consistent with numerous empirical studies on racetrack betting markets and, more generally, with the recently growing theoretical and experimental literature on non-expected utility under risk.

Though this paper has formalized an empirically supported explanation for the favorite–longshot bias by incorporating non-expected utility bettors in a game theoretical framework, it should be interesting to consider a racetrack betting market with more than two horses. Such an extension would permit to examine how the allocation of strategic bets and the magnitude of the favorite–longshot bias depend on the distribution of objective probabilities. More particularly, the empirical evidence that only extreme favorites have positive expected values need to be theoretically confirmed. As in this suggested theoretical framework bettors would face more than two possible states, one should consider rank-dependent functional forms. By doing so, there is room for finding probability weighting functions which account for these additional empirical facts. However, this remains the topic of future research.

Appendix. Proof of Theorem 1

In this appendix we provide several additional lemmas and we prove Theorem 1. Since the sequential betting game considered in this paper can be reduced to an extensive form game with perfect information, we know that there exists a subgame perfect Nash equilibrium in pure strategies (see, for example, Myerson, 1991, Theorem 4.7, p. 186). Hence, to show Theorem 1, it is sufficient to show that any outcome inconsistent with Theorem 1 is not an equilibrium outcome. We first prove the first and the third parts of the theorem.

Proof of Theorem 1(1). First note that $p < \frac{(K+1)^{1/\gamma}}{(K-t(2K+1))^{1/\gamma}+(K+1)^{1/\gamma}}$ is equivalent to $\pi(p) < \frac{K+1}{(2K+1)(1-t)}$. Assume by way of contradiction that this last inequality is satisfied and that s is an equilibrium outcome satisfying $n_F(s) \geq 1$. From Lemma 3 we know that $n_L(s) = 0$. Hence, for all $i \in F(s)$, we have:

$$\begin{array}{rrcl} & V_i(s) & > & 0 \\ \Longrightarrow & \pi(p) & > & \dfrac{n_F(s)+K}{(1-t)(n_F(s)+2K)} \\ \Longrightarrow & \pi(p) & > & \dfrac{K+1}{(1-t)(2K+1)}, \end{array}$$

a contradiction. □

Proof of Theorem 1(3). Assume that $p > \frac{(K+n)^{1/\gamma}}{(K-t(2K+n))^{1/\gamma}+(K+n)^{1/\gamma}}$, i.e., $\pi(p) > \frac{K+n}{(2K+n)(1-t)}$, and that $n_D(s) \geq 1$, where $s = s(\sigma)$, and σ is a subgame perfect equilibrium. Consider a bettor who does not bet, $i \in D(s)$. His equilibrium utility is equal to zero. If he deviates and chooses to bet on the favorite, then his utility becomes, with $s^{i-1} = (F, \ldots, F)$:

$$
\begin{aligned}
&V_i(s^{i-1}, s(\sigma \mid s^{i-1}, F)) \\
&= (1-t)\pi(p)\frac{n_F(s(\sigma \mid s^{i-1}, F)) + n_L(s(\sigma \mid s^{i-1}, F)) + 2K}{n_F(s(\sigma \mid s^{i-1}, F)) + K} - 1 \\
&\geq (1-t)\pi(p)\frac{2K+n}{K+n} - 1 > 0.
\end{aligned}
$$

Hence, σ is not an equilibrium because each bettor $i \in D(s)$ deviates and bets on the favorite. $\square$

Now, we provide several lemmas used to prove the second part of the theorem. The next lemma extends Lemma 1 to any subgame of the sequential betting game.

Lemma 4 *For any subgame (along or outside the equilibrium path), there is no equilibrium outcomes of this subgame in which some strategic bettors bet on the favorite and some strategic bettors bet on the longshot.*

Proof. The proof is similar to the Proof of Lemma 1. $\square$

Lemma 5 *Let $k \in \{1, \ldots, n-1\}$, $j \in N$, $s^{j-1} \in S^{j-1}$, and let σ be a subgame perfect equilibrium. If*

$$
p \in \left] \frac{(K+k)^{1/\gamma}}{(K-t(2K+k))^{1/\gamma} + (K+k)^{1/\gamma}}, \frac{(K+k+1)^{1/\gamma}}{(K-t(2K+k+1))^{1/\gamma} + (K+k+1)^{1/\gamma}} \right[,
$$

$n_L(s^{j-1}) = 0$*, and* $n_F(s^{j-1}) \leq k$*, then* $n_F(s(\sigma \mid s^{j-1})) \leq k$*.*

Proof. Assume by way of contradiction that $\pi(p) < \frac{K+k+1}{(1-t)(2K+k+1)}$, σ is a subgame perfect equilibrium, $n_L(s^{j-1}) = 0$, $n_F(s^{j-1}) \leq k$, and $n_F(s(\sigma \mid s^{j-1})) > k$. Let $s = s(\sigma \mid s^{j-1})$. For all $i \in F(s)$, $i \geq j$, we have

$$
\begin{aligned}
V_i(s) &= (1-t)\pi(p)\frac{n_F(s)+n_L(s)+2K}{n_F(s)+K} - 1 \\
&< \frac{K+k+1}{2K+k+1}\frac{n_F(s)+n_L(s)+2K}{n_F(s)+K} - 1 \\
&= \frac{K+k+1}{2K+k+1}\frac{n_F(s)+2K}{n_F(s)+K} - 1, && \text{because } n_L(s)=0 \text{ by Lemma 4} \\
&\leq \frac{K+k+1}{2K+k+1}\frac{2K+k+1}{K+k+1}, && \text{because } n_F(s) \geq k+1 \\
&= 0.
\end{aligned}
$$

Hence, each player $i \in F(s)$, $i \geq j$, deviates by choosing to drop from betting, which implies that s is not an equilibrium outcome. □

Lemma 6 *Let $k \in \{1, \ldots, n-1\}$. If*

$$
p \in \left]\frac{(K+k)^{1/\gamma}}{(K-t(2K+k))^{1/\gamma}+(K+k)^{1/\gamma}}, \frac{(K+k+1)^{1/\gamma}}{(K-t(2K+k+1))^{1/\gamma}+(K+k+1)^{1/\gamma}}\right[,
$$

and s is an equilibrium outcome, then $n_F(s) \geq k$.

Proof. Consider a subgame perfect equilibrium σ, and let $s = s(\sigma)$ be the associated equilibrium outcome. Assume by way of contradiction that $\pi(p) > \frac{K+k}{(1-t)(2K+k)}$ and that $n_F(s) < k$. Each bettor $i \in D(s)$ has zero utility. If he deviates and bet on the favorite, his utility becomes

$$
\begin{aligned}
& V_i(s^{i-1}, s(\sigma \mid s^{i-1}, F)) \\
&= (1-t)\pi(p)\frac{n_F(s(\sigma \mid s^{i-1}, F)) + n_L(s(\sigma \mid s^{i-1}, F)) + 2K}{n_F(s(\sigma \mid s^{i-1}, F)) + K} - 1 \\
&\geq (1-t)\pi(p)\frac{n_F(s(\sigma \mid s^{i-1}, F)) + 2K}{n_F(s(\sigma \mid s^{i-1}, F)) + K} - 1 \\
&\geq (1-t)\pi(p)\frac{2K+k}{K+k} - 1, \quad \text{because } n_F(s(\sigma \mid s^{i-1}, F)) \leq k \text{ by Lemma 5} \\
&> \frac{K+k}{(2K+k)}\frac{2K+k}{K+k} - 1 = 0.
\end{aligned}
$$

Hence, σ is not an equilibrium because each bettor $i \in D(s)$ deviates and bets on the favorite. $\square$

From Lemma 5 and 6 we know that the number of bets on the favorite is in accordance with Theorem 1(2). It remains to show that bets on the favorite are always done by the first bettors.

Lemma 7 *Let $k \in \{1, \ldots, n-1\}$. If*

$$p \in \left] \frac{(K+k)^{1/\gamma}}{(K-t(2K+k))^{1/\gamma}+(K+k)^{1/\gamma}}, \frac{(K+k+1)^{1/\gamma}}{(K-t(2K+k+1))^{1/\gamma}+(K+k+1)^{1/\gamma}} \right[,$$

$s_i = D$ and $s_j = F$, where $j > i$, then s is not an equilibrium outcome.

Proof. From Lemma 5 and 6 we have $n_F(s) = k$. Consider a bettor $i \in D(s)$ such that $s_j = F$ for some $j > i$. This implies that $n_F(s^{i-1}) < k$, and thus $n_F(s(\sigma \mid s^{i-1}), F) \leq k$ from Lemma 5. In that case it can be shown as in the Proof of Lemma 6 that bettor i deviates by betting on the favorite. This completes the proof of the lemma and of Theorem 1. $\square$

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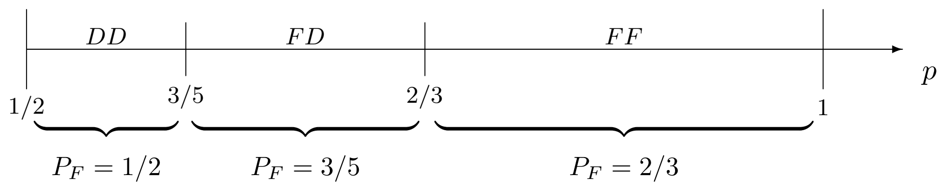

Figure 1: Equilibrium outcomes and subjective probabilities of the favorite when $n = K = 2$, $\gamma = 1$, and $t = 0$.

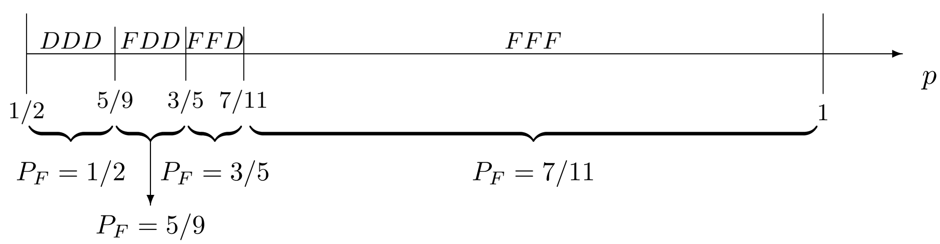

Figure 2: Equilibrium outcomes and subjective probabilities of the favorite when $n = 3$, $K = 4$, $\gamma = 1$, and $t = 0$.

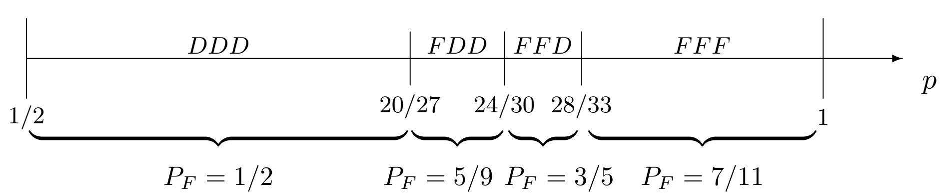

Figure 3: Equilibrium outcomes and subjective probabilities of the favorite when $n = 3$, $K = 4$, $\gamma = 1$, and $t = 0.25$.

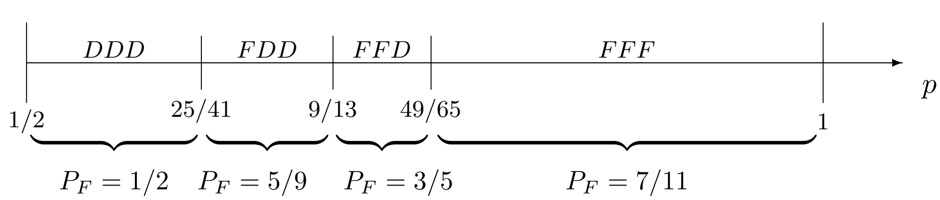

Figure 4: Equilibrium outcomes and subjective probabilities of the favorite when $n = 3$, $K = 4$, $\gamma = 1/2$, and $t = 0$.