Partial Certifiability and Information Precision in a Cournot Game *

Frédéric KOESSLER†

January 9, 2002

Abstract

This paper examines strategic information revelation in a Cournot duopoly with incomplete information about firm 1's cost *and* information precision. Firm 2 relies on certifiable and ex post submissions of firm 1, without necessarily knowing whether firm 1 knows its cost or not. The sequential equilibria of the induced communication game are determined for different certifiability possibilities. A perfectly revealing equilibrium in which information precision is irrelevant is obtained under full certifiability. On the contrary, it is shown that if only payoff-relevant (fundamental) events can be certified, then the equilibrium output and profit of firm 1 decreases with its average information precision if this firm is uninformed or if its cost is high. A consequence of this local effect is that information precision has, *on average*, no value for a firm.

KEYWORDS: Strategic information revelation; Information precision; Cournot competition; Cost uncertainty; Higher order uncertainty.

JEL CLASSIFICATION: C72; D43; D82; L13.

*Helpful comments were provided by Rodolphe Dos Santos Ferreira, Hubert Stahn, Gisèle Umbhauer, and Anthony Ziegelmeyer.

†Department of Economics, BETA–*Theme*, Université Louis Pasteur, 61 Avenue de la Forêt-Noire, F-67085 Strasbourg Cedex (France).
Tel.: (33) 3 90 24 20 90.
E-mail: koessler@cournot.u-strasbg.fr.
URL: http://cournot.u-strasbg.fr/koessler.

1 Introduction

The purpose of this paper is to examine firms' disclosure choices and inferences in an industry with uncertainty concerning the quality of firms' technology and information precision. While it is well known that the revelation of information—and hence, the equilibrium level of outputs—drastically depends on which type of information can be disclosed, the effects of competitors' uncertainty concerning others' information precision remain unexplored. This uncertainty about information precision, measured as the frequency of access to fundamental information, can vary for various reasons. For example, firms can face and observe different technological shocks, they may have different successes to technological innovations, they may have uncertain and different tax rates due to their stochastic polluting activities, etc. Thus, they might have different and not commonly known access to information about their production costs. Alternatively, firms established in a market may not know newcomers' technology and may not know whether recently established firms perfectly know their own technology.

To conduct our analysis, we consider a simple model of Cournot competition with two firms, in which firm 1 observes private—possibly imperfect—information about the state of its technology. For simplicity, we assume that either firm 1 perfectly learns its cost, or receives no information about it. In this latter case, the only "information" of firm 1 is that it does not know its cost. A priori, this information is not received by firm 2, although it can be received in a latter stage from firm 1's voluntary disclosures. Hence, before the communication stage, firm 2 does not know whether its rival is informed about its cost or not. The probability $\gamma \in [0, 1]$ that firm 1 receives an informative signal characterizes firm 2's uncertainty concerning firm 1's information precision. Since this probability is known by firm 2, the average precision of firm 1's information is common knowledge, contrary to its form (the informativeness about the cost) and, of course, its content (the cost itself).

After observing its private signal, firm 1 chooses whether to disclose and certify some information to the other firm. We will assume that revealed information is always certified (proved, or verifiable) and true. Such an assumption is common in many papers dealing with strategic information revelation (Grossman, 1981; Milgrom, 1981; Okuno-Fujiwara, Postlewaite, and Suzumura, 1990; Shin, 1994; Lipman and Seppi, 1995; Glazer and Rubinstein, 2001, is not an exhaustive list). It can be justified in various economic or legal contexts, and it highlights several economic and behavioral phenomena which cannot be obtained otherwise. In particular, it illustrates how agents update their beliefs when others remain silent.[1] Moreover, the restriction to truthful certified reports is compensated by the fact that all information might not be certified. This certifiability configuration is made explicit in the model. Finally, information revelation remains voluntary, and might be strategically partial and vague. In short, firms cannot lie but may try to withhold their information by remaining silent or by revealing only favorable (albeit truthful) information.

In the model presented here, the following types of certifiability possibilities are considered. In one extreme configuration, neither firm 1's cost nor its information precision can be certified. Of course, communication is irrelevant in such a setting. The other extreme configuration allows everything to be certified. In between, the most realistic configuration corresponds to the one in which firm 1 can certify its cost when it knows it, but is not able to certify that it is uninformed. This is possible, for example, if firms can conduct laboratory experiments that eventually reveal their technology and costs, but are not able to certify that the laboratory experiment was unsuccessful.

In a second stage, the Cournot game in played according to the messages delivered in the communication stage. In this second-stage game, each firm chooses its level of output given its initial information and the information revealed in the first-stage game. These output levels jointly determine the market price according to the inverse demand function. Our approach concerning the revelation of information is essentially the one of Okuno-Fujiwara et al. (1990). Contrary to models of information sharing among oligopolists (as, e.g., Novshek and Sonnenschein, 1982; Clarke, 1983; Vives, 1984; Gal-Or, 1985; Shapiro, 1986; Malueg and Tsutsui, 1996, 1998) we consider ex post incentives to share information, i.e., communication choices are made at the interim stage, once specific information is received. This feature is usually at variance with ex ante calculations which require binding contracts or commitments, or a central agency ensuring the collect of market data and the revelation of information. For example, firms may commit not to share information, but once signals are received a firm may want to convince the rival that his own cost is low to induce soft behavior on its part.

Under the assumption that all information can be certified, the only sequen-

[1] Actually, in interactive decision situations considered in this literature, cheap talk communication (communication where messages are non-binding, have no costs associated, and are not certifiable) is not informative and cannot matter. The reason is that in a situation in which talk is cheap, a firm would always want its rival to believe that it has the lowest costs.

tial equilibrium of the two-stage game is shown to be perfectly revealing, whatever the prior probabilities. Nevertheless, when only the cost (i.e., the fundamental and physical characteristics of the economy) can be certified, and as long as firm 2 is not sure that firm 1 is perfectly informed about its cost, the only sequential equilibrium is a partially revealing equilibrium. In this particularly interesting configuration, we analyze the effects of firm 1's average information precision. This study is conducted in the same line as Shin (1994), who investigated a decision maker's inferences and decision rules in presence of uncertainty concerning the precision of the informed parties' information. In short, he showed that the decision maker becomes more skeptical when he knows that the average precision of the informed parties' information increases. A very similar phenomenon is obtained in our model. More precisely, our main result is that the precision of firm 1's information has two effects: an average (global) effect, and a local effect. First, when firm 1's average information precision increases, more information is revealed, on average. In the limiting case in which $\gamma = 1$, all information is revealed, i.e., firm 2 perfectly learns firm 1's cost. On the contrary, when $\gamma = 0$, firms are symmetrically informed and neither knows firm 1's cost. Hence, no information is revealed. Second, when the average precision of firm 1's information increases, firm 2 becomes more skeptical about firm 1's technology, resulting in an increase of its production and profits when firm 1 conceals its information. However, it is shown that, on average, these effects cancel each other out. Hence, on average, the frequency of informative signals has no effect. Said differently, *average* productions and profits of both firms do not depend on γ.

The intuition of the effects of firm 1's average information precision on firm 2's inferences is relatively simple. First, when firm 1 reveals its low cost to the competitor, it reduces the competitor production. Hence, since outputs are strategic substitutes, firm 1's output and profit increase. Second, when firm 1 has a high cost, it has no incentive to reveal it since it will induce the competitor to "overproduce". Consequently, firm 1 will have an incentive to reveal its cost if and only if this cost is low. Knowing that the cost is revealed if and only if it is low, there are two possibilities when firm 2 does not receive information from firm 1: either firm 1 is genuinely uninformed, or its cost is high. When the average precision of firm 1's information increases, i.e., when there are less uncertainties about firm 1's information, the possibility that the cost is high becomes more likely. Therefore, firm 2 puts more weight on the alternative that firm 1 knows that its cost is high, i.e., it becomes more skeptical about firm 1's cost. As a result, when nothing has been revealed, firm 2's production and profit increase, whereas firm 1's production and profit decrease.

The paper is organized as follows. In Section 2, we present the Cournot game. In Section 3, we define the communication game and associated sequential equilibria. These equilibria in presence of various information structures and certifiability possibilities are analyzed in Sections 4 and 5. In Section 6 we study the effects of information precision. Finally, we conclude in Section 7. Proofs of the propositions can be found in the Appendix.

2 The Cournot Game

We consider a duopoly playing Cournot competition in which each firm $i \in N = \{1, 2\}$ can produce according to a constant marginal cost λ_i. The two firms produce identical products and face a linear (inverse) demand function $p(q_1 + q_2) = a - b(q_1 + q_2)$, where $q_i \in \mathbb{R}_+$ is firm i's output and a, $b > 0$ are parameters. The utility (payoff) of each firm i is

$$\begin{aligned} u_i(q_1, q_2) &= \frac{1}{b}(p(q_1 + q_2)q_i - \lambda_i q_i) \\ &= q_i(-\theta_i - q_1 - q_2), \end{aligned}$$

where $\theta_i = \frac{\lambda_i - a}{b}$. We call θ_i firm i's cost, even if θ_i is an affine and increasing transformation of the cost λ_i. We assume for simplicity that $\theta_2 = -1$ and $\theta_1 \in [-2, -1/2]$. Firms' reaction functions are

$$\begin{aligned} q_1(q_2, \theta_1) &= \frac{-\theta_1 - q_2}{2}, \\ q_2(q_1) &= \frac{1 - q_1}{2}. \end{aligned}$$

Hence, in the Nash equilibrium, firms' outputs are given by the vector

$$q^N(\theta_1) = (q_1^N(\theta_1), q_2^N(\theta_1)),$$

such that

$$q_1^N(\theta_1) = \frac{-1 - 2\theta_1}{3} \ (\in [0,1]),$$
$$q_2^N(\theta_1) = \frac{2 + \theta_1}{3} \ (\in [0, \frac{1}{2}]). \quad (1)$$

Associated payoffs are $u_i^N(\theta_1) = (q_i^N(\theta_1))^2$, for $i \in N$ and $\theta_1 \in [-2, -1/2]$. For example, if $\theta_1 = \theta_2 = -1$ (firms are symmetric), then $q_1 = q_2 = \frac{1}{3}$. If $\theta_1 = -\frac{1}{2}$ (firm 1 has a high cost), then firm 1 does not produce, and firm 2 is a monopoly producing $q_2 = \frac{1}{2}$. Finally, if $\theta_1 = -\frac{3}{2}$ (firm 1 has a low cost), then $q_1 = \frac{2}{3}$ and $q_2 = \frac{1}{6}$. More generally, an increase in θ_1 decreases firm 1's output, increases firm 2's output, and increases the equilibrium market price.

Figure 1 represents firms' reaction functions and the Nash equilibria depending on firm 1's cost.

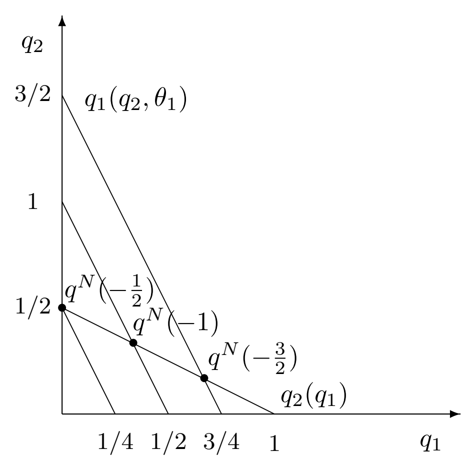

Figure 1: Nash Equilibrium Outputs with Complete Information.

We introduce incomplete information by assuming that θ_1 is determined by a state of the world $\omega \in \Omega$. Firm 2's cost $\theta_2 = -1$ remains common knowledge throughout the analysis. The cost of firm 1 can be low or high, i.e., $\theta_1 \in \Theta_1 = \{\theta_1^L, \theta_1^H\}$, where $-\frac{1}{2} \geq \theta_1^H > \theta_2 = -1 > \theta_1^L \geq -2$.[2] This cost is determined by a function $\tau : \Omega \to \Theta_1$. We denote by p the probability distribution over Ω, where $p(\omega) > 0$ for all $\omega \in \Omega$. The partition of firm 1 over Ω is denoted by H_1. An information set $h_1(\omega) \in H_1$ contains all states firm 1 considers as possible when the

[2]To simplify the exposition, we assume that firm 1 has only two possible cost realizations. The analysis extends obviously to several levels of costs in the interval $[-2, -1/2]$.

real state is $\omega \in \Omega$. We will always assume that firm 2 has no a priori information about Ω, i.e., its partition is $H_2 = \{\Omega\}$. The prior probability that firm 1 has a low cost is given by any $\beta \in]0,1[$, and will never be specified throughout the analysis. Utility functions are extended to ${\mathbb{R}_+}^2 \times \Omega$ by $u_i(q_1, q_2, \omega) \equiv q_i(-\tau(\omega) - q_1 - q_2)$, for all $i \in \{1,2\}$. Hence, firm i's expected utility at ω is given by $E_p(u_i(q_1, q_2, \cdot) \mid h_i(\omega))$.

3 Certifiability and Communication

In this section we define the sequential equilibria of the two-stage game according to the information structure and certifiability possibilities. In the communication stage, a *report* (or message) by firm 1 is a nonempty subset $x \subseteq \Omega$, interpreted as an assertion by the firm that the state of the world belongs to x. We denote by $X(\omega)$ the set of reports that can be certified by the firm when the real state of the world is ω. The constraint that the firm must report truthfully implies that for all $\omega \in \Omega$ and $x \in X(\omega)$ we have $h_1(\omega) \subseteq x$. In other words, firm 1 can only reveal events it knows. The set of all certifiable reports is denoted by $\mathcal{X} = \bigcup_{\omega \in \Omega} X(\omega)$. Note that it is sufficient to consider reports which are an union of firm 1's information sets, i.e., $\mathcal{X}$ is a subset of the σ-algebra generated by firm 1's partition H_1. A *communication strategy* is a H_1 measurable function $c : \Omega \to \mathcal{X}$ with the property that $c(\omega) \in X(\omega)$ for all $\omega \in \Omega$. When all truthful messages can be sent, then firm 1's report can be very precise concerning its information, as when $c(\omega) = h_1(\omega)$, but it can also be very vague, as when $c(\omega) = \Omega$.

Second-stage strategies specify the quantities produced by the firms after the communication stage. Formally, a *second-stage* or *payoff-relevant strategy* of firm 1 is a function $\sigma_1 : \mathcal{X} \times \Omega \to \mathbb{R}_+$ such that $\sigma_1(x, \cdot) : \Omega \to \mathbb{R}_+$ is measurable with respect to H_1 for all $x \in \mathcal{X}$. A second-stage strategy of firm 2 is a function $\sigma_2 : \mathcal{X} \to \mathbb{R}_+$. Thus, $\sigma_1(x, \omega)$ and $\sigma_2(x)$ are the quantities produced by the firms at ω when firm 1 has reported x. Given a report x, let $I(x)$ be an nonempty subset of x representing the *conjecture* reached by firm 2 concerning firm 1's cost. This means that when firm 1 reveals x, firm 2 will conclude that $\omega \in I(x)$. It is assumed that $I(x) = \bigcup_{\omega \in I(x)} h_1(\omega)$, i.e., the conjecture made by firm 2 is consistent with firm 1's information.

For the communication game presented here, a sequential equilibrium is a triple

$$((\sigma_i)_{i \in N}, c, I),$$

satisfying the following conditions:

(i) For every state $\omega \in \Omega$ and every possible message $x \in X(\omega)$, $\sigma_1(x, \omega)$ solves

$$\max_{q_1 \in \mathbb{R}_+} E_p(u_1(q_1, \sigma_2(x), \cdot) \mid h_1(\omega)),$$

and $\sigma_2(x)$ solves

$$\max_{q_2 \in \mathbb{R}_+} E_p(u_2(\sigma_1(x, \cdot), q_2, \cdot) \mid I(x)).$$

(ii) For every state $\omega \in \Omega$, $c(\omega)$ solves

$$\max_{x \in X(\omega)} E_p(u_1(\sigma_1(x, \cdot), \sigma_2(x), \cdot) \mid h_1(\omega)).$$

(iii) For every x in the range of c, $I(x) = c^{-1}(x)$.

Condition (i) states that firms' outputs are part of a Bayesian equilibrium of the continuation Bayesian games generated by communication. That is, strategies are optimal in the Cournot competition contingent on the information exchanged. According to condition (ii), firm 1 reveals an optimal message given the continuation Bayesian equilibria. Finally, condition (iii) is the "rational expectation" condition, stating that firm 2 uses Bayes' rule to update its belief along the equilibrium path.

In the following sections we analyze the Bayesian-Nash equilibria as well as strategic information revelation (through the sequential equilibrium defined above) for different kinds of information structures and certifiability possibilities.

4 Perfect Information Precision

To begin with, we consider the simplest incomplete information structure where $\Omega = \{\omega_1, \omega_2\}$, $H_1 = \{\{\omega_1\}, \{\omega_2\}\}$, $H_2 = \{\{\omega_1, \omega_2\}\}$, $\tau(\omega_1) = \theta_1^L$, and $\tau(\omega_2) = \theta_1^H$. This means that firm 1 always knows its cost, firm 2 does not know firm 1's cost, and this is common knowledge. This is the standard form of asymmetric information of economic models. It incorporates asymmetry about fundamentals, not about others' knowledge.

Let $\beta = p(\omega_1)$ be the probability that firm 1's cost is low and let $\theta_1^M = \beta\theta_1^L + (1 - \beta)\theta_1^H$ be the average of its cost. This incomplete information game is essentially a version of Okuno-Fujiwara et al. (1990, Example 1) without specific prior probabilities.[3] In the Bayesian equilibrium, best responses are determined by

$$q_1(q_2,\omega) \in \arg\max_{q_1} \{q_1(-\tau(\omega) - q_1 - q_2)\},$$
$$q_2(q_1^M) \in \arg\max_{q_2} \big\{\beta q_2\big(1 - q_1(\omega_1) - q_2\big) + (1-\beta)\, q_2\big(1 - q_1(\omega_2) - q_2\big)\big\},$$

where $q_1^M = \beta q_1(\omega_1) + (1-\beta)q_1(\omega_2)$ is the expected output of firm 1. We obtain the following best responses:

$$q_1(q_2,\omega) = \frac{-\tau(\omega) - q_2}{2},$$
$$q_2(q_1^M) = \frac{1 - q_1^M}{2}.$$

Then, at the Bayesian equilibrium the (information-contingent) quantities produced by each firm are given by the vector $q^B = (q_1^B(\omega_1), q_1^B(\omega_2), q_2^B)$ satisfying

$$q_1^B(\omega_1) = \frac{-2 - \theta_1^M - 3\theta_1^L}{6}$$
$$q_1^B(\omega_2) = \frac{-2 - \theta_1^M - 3\theta_1^H}{6}$$
$$q_2^B = \frac{2 + \theta_1^M}{3},$$

and firm 1's utility at $\omega \in \Omega$ is $u_1^B(\omega) = (q_1^B(\omega))^2$. Hence, the output of firm 2 is increasing in the expected cost of its rival, while the output of firm 1 is decreasing with its own cost. This equilibrium is best explained by referring to Figure 2 on the next page, which represents firms' reaction functions and the Bayesian-Nash equilibrium outputs (E and F) depending on firm 1's cost. One can easily verify, using θ_1^L, $\theta_1^H \in [-2, -\frac{1}{2}]$, that

[3] Actually, Okuno-Fujiwara et al. (1990) provided sufficient conditions for the existence of perfectly revealing equilibria which are satisfied in this section when low cost can be certified. Hence, the proof of the first part of Proposition 1 is needless.

$$u_1^N(\theta_1^L) > u_1^B(\omega_1),$$
$$\text{and} \quad u_1^N(\theta_1^H) < u_1^B(\omega_2).$$

As an intuitive consequence we might expect that firm 1 would like to convince the rival that it has low cost by revealing its cost when it is low (at ω_1) but not when it is high (at ω_2). Consequently, there should be a perfectly revealing sequential equilibrium of the communication game as long as low cost can be certified. Graphically, when firm 1 reveals its cost when it is low, we move from the original outcome E to the outcome E' in Figure 2. Hence, firm 2 contracts its output to the benefit of firm 1 and the price increases since the combined output decreases. Therefore, when the cost is low, firm 1 has an incentive to reveal its cost. On the contrary, firm 1 has no incentive to reveal its cost when it is high. This can be seen in Figure 2 because a move from F to F' is unfavorable to firm 1. The next proposition asserts that the unique equilibrium is indeed perfectly revealing as long as firm 1 can certify that its cost is low. If not, the only equilibrium is non-revealing.

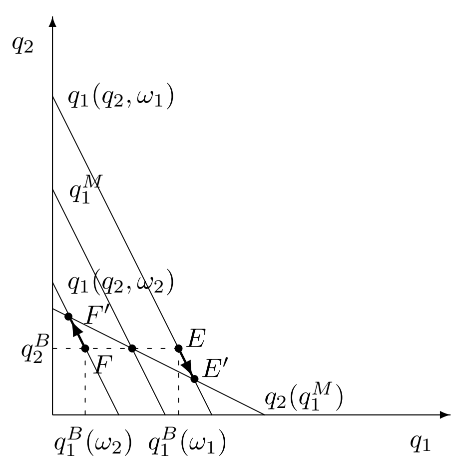

Figure 2: Bayesian-Nash Equilibrium Outputs with Incomplete Information.

Proposition 1 *Assume that firm 1 always knows its cost and that firm 2 does not know it.*

1. *If firm 1 can certify its cost when it is low (i.e., $\{\omega_1\} \in \mathcal{X}$) then the unique sequential equilibrium is perfectly revealing.*

2. *If firm 1 cannot certify its cost when it is low (i.e., $\{\omega_1\} \notin \mathcal{X}$) then the unique sequential equilibrium is non-revealing.*

We see that firm 1 with a high cost is better off when the other firm does not know it. On the contrary, firm 1 with a low cost prefers that firm 2 knows it (in order to induce firm 2 to produce less). Therefore, when its cost is low firm 1 reveals it. When firm 2 observes that firm 1 remains silent, firm 2 deduces that the cost is high. Hence, the unravelling argument applies once again in this context.

However this argument fails when firm 1 does not know its cost in some states and cannot certify that it does not know it. In this case, when nothing is revealed by firm 1, firm 2 cannot deduce that the cost is high, leading to a failure of perfect revelation. The next section shows this phenomena, in the same line as Okuno-Fujiwara et al. (1990, Example 3), but with arbitrary prior probabilities and arbitrary information precision for firm 1. Hence, our framework allows us to investigate the effect of firm 1's information precision on firm 2's inferences.

5 Partial Information Precision

We consider the same framework as in the previous section, except that we allow the possibility that firm 1 might not know its own cost. There are four states of the world, and the initial information structure is $H_1 = \{\{\omega_1\}, \{\omega_2\}, \{\omega_3, \omega_4\}\}$ and $H_2 = \{\Omega\}$, where $\tau(\omega_1) = \tau(\omega_3) = \theta_1^L$ and $\tau(\omega_2) = \tau(\omega_4) = \theta_1^H$. That is, firm 1 knows its cost at ω_1 and ω_2, but does not know it at ω_3 and ω_4. Firm 2 does not know whether firm 1 knows its cost or not, and this is common knowledge between both firms. The *payoff-relevant certifiability level* is characterized by a set of messages $\mathcal{X} = \{\{\omega_1\}, \{\omega_2\}, \Omega\}$. This means that only payoff-relevant events can be certified: the event $\{\omega_3, \omega_4\} = \Omega \backslash \{\omega : h_1(\omega) \subseteq \{\omega_1, \omega_2\}\}$ (firm 1 does not know its cost) cannot be certified. The payoff-relevant certifiability level is reasonable since it does not permit the difficult task consisting in certifying that one does not know something. As will be seen, allowing or not payoff-relevant information to be revealed will drastically modify equilibrium outcomes.

Denote by $\gamma \in]0, 1[$ the probability that firm 1 is informed about its cost, i.e., $p(\{\omega_1, \omega_2\}) = \gamma$. As before, β is the probability that firm 1's cost is low, i.e., $p(\{\omega_1, \omega_3\}) = \beta$. Then, prior probabilities of the states of the world are

$$
\begin{array}{rclcrcl}
p(\omega_1) & = & \beta\gamma & & p(\omega_3) & = & \beta(1-\gamma) \\
p(\omega_2) & = & (1-\beta)\gamma & & p(\omega_4) & = & (1-\beta)(1-\gamma).
\end{array}
$$

With this new information structure, the Bayesian equilibrium is determined by

$$
\begin{aligned}
q_1(q_2,\omega) &\in \arg\max_{q_1} \{q_1(-\tau(\omega)-q_1-q_2)\}, \text{ if } \omega = \omega_1,\, \omega_2, \\
q_1(q_2,\omega_3) = q_1(q_2,\omega_4) &\in \arg\max_{q_1} \left\{\beta q_1(-\theta_1^L - q_1 - q_2) + (1-\beta)q_1(-\theta_1^H - q_1 - q_2)\right\} \\
&= \arg\max_{q_1} \left\{q_1(-\theta_1^M - q_1 - q_2)\right\}, \\
q_2(q_1^M) &\in \arg\max_{q_2} \sum_{\omega\in\Omega} p(\omega) q_2\big(1 - q_1(\omega) - q_2\big),
\end{aligned}
$$

yielding to the following (information-contingent) best responses:

$$
q_1(q_2,\omega) = \frac{-\tau(\omega)-q_2}{2}, \text{ if } \omega = \omega_1,\ \omega_2, \tag{2}
$$

$$
q_1(q_2,\omega_3) = q_1(q_2,\omega_4) = q_1^M = \frac{-\theta_1^M - q_2}{2}, \tag{3}
$$

$$
q_2(q_1^M) = \frac{1-q_1^M}{2},
$$

where, as before, $\theta_1^M = \beta\theta_1^L + (1-\beta)\theta_1^H$.[4] At the Bayesian equilibrium, outputs are quite similar to those of the previous section:

$$
\begin{aligned}
q_1^B(\omega_1) &= \frac{-2-\theta_1^M - 3\theta_1^L}{6}, \\
q_1^B(\omega_2) &= \frac{-2-\theta_1^M - 3\theta_1^H}{6}, \\
q_1^B(\omega_3) = q_1^B(\omega_4) &= \frac{-1-2\theta_1^M}{3}, \\
q_2^B &= \frac{2+\theta_1^M}{3}.
\end{aligned}
$$

Associated payoffs of firm 1 at $\omega \in \Omega$ is $u_1^B(\omega) = \left(q_1^B(\omega)\right)^2$. The next proposition shows the influence of certifiability on the incentive to reveal information, and thus

[4] Notice that since $q_1(q_2,\omega_3)$ or $q_1(q_2,\omega_4)$ is an average of firm 1's best response at ω_1 and ω_2, $q_2(q_1)$ is also the best response of firm 2 when firm 2 knows that firm 1 knows its cost.

on equilibrium outputs.

Proposition 2 *Assume that firm 2 does not know whether firm 1 knows its cost or not, and that firm 2 does not know firm 1's cost.*

1. *If firm 1 can certify its cost and that it does not know its cost, i.e.,*

$$\mathcal{X} = \{\{\omega_1\}, \{\omega_2\}, \{\omega_3, \omega_4\}, \Omega\},$$

   then the unique sequential equilibrium is perfectly revealing. Hence, outputs and utilities are the same as those with complete information.

2. *With the payoff-relevant certifiability level (i.e., $\mathcal{X} = \{\{\omega_1\}, \{\omega_2\}, \Omega\}$) the unique sequential equilibrium is partially revealing: firm 2 learns the cost iff it is low (and firm 1 knows it). In this partially revealing equilibrium outputs and utilities are*

$$\begin{aligned}
q_1^P(\omega_1) &= q_1^N(\theta_1^L), \\
q_1^P(\omega_2) &= -\frac{1}{3} + \frac{(\beta(1+3\gamma)-4)\theta_1^H - \beta(1-\gamma)\theta_1^L}{6(1-\beta\gamma)}, \\
q_1^P(\omega_3) = q_1^P(\omega_4) &= -\frac{1}{3} + \frac{(1-\beta)(3\gamma\beta-4)\theta_1^H + \beta(\gamma(1+3\beta)-4)\theta_1^L}{6(1-\beta\gamma)}, \\
q_2^P(\omega_1) &= q_2^N(\theta_1^L), \\
q_2^P(\omega) &= \frac{2}{3} + \frac{(1-\beta)\theta_1^H + \beta(1-\gamma)\theta_1^L}{3(1-\beta\gamma)}, \text{ if } \omega \neq \omega_1,
\end{aligned}$$

   and $u_i^P(\omega) = (q_i^P(\omega))^2$, for all $i \in N$ and $\omega \in \Omega$.

3. *If firm 1 cannot certify its cost when it is low (i.e., $\{\omega_1\} \notin \mathcal{X}$) then firm 2 keeps its prior probabilities about firm 1's cost. Hence, outputs and utilities are the same as those with incomplete information.*

6 Inferences and Outcomes at the Partially Revealing Equilibrium

In this section we analyze into more details the partially revealing equilibrium found in the second part of Proposition 2. We consider the states of the world $\omega \neq \omega_1$ since at ω_1 we get the same outputs and beliefs as in the Nash equilibrium with complete information. Hence, we consider the states in which firm 1 reveals nothing. In this case, firm 2 considers as possible only the states $\omega \neq \omega_1$. By denoting μ its belief about firm 1's cost $\theta_1 \in \{\theta_1^L, \theta_1^H\}$ and by using Bayes' rule, we get

$$\begin{aligned} \mu(\theta_1^L) &= p(\omega_3 \mid \{\omega_2, \omega_3, \omega_4\}) &= \frac{\beta(1-\gamma)}{1-\beta\gamma} \\ \mu(\theta_1^H) &= p(\{\omega_2, \omega_4\} \mid \{\omega_2, \omega_3, \omega_4\}) &= \frac{1-\beta}{1-\beta\gamma}. \end{aligned}$$

Therefore, when the precision of firm 1's information increases (γ increases), firm 2 puts more weight on θ_1^H. In other words, when the precision of firm 1's information increases, firm 2 is more skeptical about firm 1's technology when this latter remains silent. Hence, its beliefs that firm 1 has a high cost increases, resulting in more production by firm 2 and less production by firm 1. We get the main result of this paper:

Proposition 3 *Assume that firm 2 does not know whether firm 1 knows its cost or not, that firm 2 does not know firm 1's cost, and that only fundamental (payoff-relevant) information can be revealed by firm 1. Then, at equilibrium, if nothing has been revealed by firm 1 (i.e., either firm 1 has a high cost or does not know its cost):*

1. Firm 2's belief $\mu(\theta_1^L)$ that firm 1's cost is low is decreasing with firm 1's information precision γ. Firm 2's belief $\mu(\theta_1^H)$ that firm 1's cost is high is increasing with firm 1's information precision γ.

2. Firm 1's output and profit are decreasing with firm 1's information precision γ, whereas firm 2's output and profit are increasing with firm 1's information precision.

The last proposition states that information precision is unfavorable to firm 1 in states of the world in which it has no incentive to communicate. What is the average effect of firm 1 information precision on firms' production and profits? To answer this question we have to compute how average productions and profits change with the probability γ that firm 1 is informed. From the second part of Proposition 2, average outputs at the partially revealing equilibrium are

$$E_p(q_1^P(\cdot)) = \sum_{\omega\in\Omega} p(\omega)\, q_1^P(\omega) = \frac{-1-2\theta_1^M}{3},$$
$$E_p(q_2^P(\cdot)) = \sum_{\omega\in\Omega} p(\omega)\, q_2^P(\omega) = \frac{2+\theta_1^M}{3}.$$

That is, average outputs are the same as those with complete information. This is interesting since it implies that the average access of information of firm 1 has no effect, on average, on its profit. Hence, even if the firm has, on average, better access to information, it does not improve its profit: for any $\gamma \in [0, 1]$, its average profit is the same. The reason for this phenomenon is due to two opposite effects. A positive and global effect of an increase of the frequency of information precision for firm 1 results from the fact that, when its cost is low, it can certify it more often. However, at the same time, when its cost is high and it knows it, or when it does not know its cost, no revelation will result in more skeptical beliefs and thus more production by firm 2. On average, as shown, these effects cancel each other out, for any $\gamma \in [0, 1]$.

7 Concluding Remarks

In this paper we have provided several results useful for understanding the role of higher-order uncertainty in Cournot games with information revelation possibilities. For example, we have shown that an increase of a firm's average access to fundamental information about its cost has two effects when information revelation is possible. On the one hand, an increase of the firm's average information precision enables this firm to convince more frequently its competitor that it has a low cost. On the other hand, a more subtle and indirect effect results in more skeptical beliefs from the competitor in the face of silence. Hence, when firm 1 conceals its information (i.e., when its cost is high or when the firm does not know its cost) an increase of the firm average information precision reduces its production and profit to the advantage of its rival. As shown, these effects cancel each other out on average. This might imply that if a firm has the possibility to invest in costly R&D expenses to *learn* its technology (not to improve it), it will not do so since on average, there is no gain from more precise information.

There are various ways in which our analysis can be extended. One possibility is to consider a more general information structure, as in Shin (1994), in which there are more than two possibilities of information precision. Indeed, in the Cournot game presented here, firm 1 is either perfectly informed about its cost or not informed at all. However, the qualitative results will remain the same. An increase of the average precision of firm 1 will lead to greater skepticism in the face of vagueness. An other possibility is to consider more than two firms, each one having private information about its cost, and various access to information. Some other interesting phenomena might emerge, although we expect that firms' inference should remain qualitatively similar. From our point of view, more interesting extensions should be to consider more general demand functions, or to consider uncertainty about others' information precision concerning other parameters of the industry, like the slope or intercept of demand. There are various concrete contexts in which firms do not know the exact estimate of other firms about the state of demand in their sector. An analysis of incentives to disclose information should be fruitful in such contexts.

Appendix. Proofs

Proof of Proposition 1. We first show the second part of the proposition. Assume that $\mathcal{X} = \{\{\omega_2\}, \Omega\}$ (the result is obvious if $\mathcal{X} = \{\Omega\}$). If the sequential equilibrium is perfectly revealing then $c_1(\omega_2) = \{\omega_2\}$ and $c_1(\omega_1) = \Omega$. Firm 1's payoff at ω_2 is $u_1^N(\theta_1^H) = (\frac{1+2\theta_1^H}{3})^2$. If firm 1 deviates, i.e., reveals Ω, then firm 2 knows (wrongly) $\{\omega_1\}$ by Bayesian updating, and thus firm 2 plays $q_2^N(\theta_1^L)$. Playing rationally against $q_2^N(\theta_1^L)$, firm 1's payoff becomes $u_1(\frac{-\theta_1^H - q_2^N(\theta_1^L)}{2}, q_2^N(\theta_1^L), \omega_2) = (\frac{2+\theta_1^L+3\theta_1^H}{6})^2 > (\frac{1+2\theta_1^H}{3})^2$ since $-1/2 \geq \theta_1^H > \theta_1^L$. Therefore, firm 1 deviates from full revelation, and the perfectly revealing communication strategy is not an equilibrium. On the contrary, if $c_1(\omega_2) = \Omega$, then by deviating at ω_2 firm 1 gets $u_1^N(\theta_1^H)$ by the certifiability constraint, which is smaller than $u_1^B(\omega_2)$.

Now, to show the first part of the proposition, assume that $\{\omega_1\} \in \mathcal{X}$, i.e., firm 1 can certify its cost when it is low. We begin to show that there is a perfectly revealing equilibrium where $c_1(\omega_1) = \{\omega_1\}$ and $c_1(\omega_2) = \Omega$, i.e., firm 1 reveals its cost only if it is low.[5] In this case the conjecture of firm 2 is uniquely defined ($I(\{\omega\}) = \{\omega\}$ for all ω and $I(\Omega) = \{\omega_2\}$). Rational communication is verified if firm 1 does not deviate when its cost is low (at ω_2 firm 1 cannot modify the information structure),

[5]There is also a perfectly revealing equilibrium in which $c_1(\omega) = \{\omega\}$ for all $\omega \in \Omega$.

i.e., if $u_1^N(\theta_1^L) \geq u_1(\frac{-\theta_1^L - q_2^N(\theta_1^H)}{2}, q_2^N(\theta_1^H), \omega_1)$, which is equivalent to $(\frac{1+2\theta_1^L}{3})^2 \geq (\frac{2+\theta_1^H+3\theta_1^L}{6})^2$. This last inequality is verified since $-1/2 \geq \theta_1^H > \theta_1^L$.

To show that all sequential equilibria are perfectly revealing, assume on the contrary that $c_1(\omega_1) = c_1(\omega_2) = \Omega$. We immediately see that firm 1 deviates at ω_1 by revealing $\{\omega_1\}$ since $u_1^N(\theta_1^L) > u_1^B(\omega_1)$. This completes the proof. $\square$

Proof of Proposition 2.

1. To show that there exists a perfectly revealing equilibrium, let $c_1(\omega_1) = \{\omega_1\}$, $c_1(\omega_2) = \Omega$, $c_1(\omega_3) = c_1(\omega_4) = \{\omega_3, \omega_4\}$, $I(x) = \{\omega_2\}$ if $\omega_2 \in x$, $I(x) = \{\omega_3, \omega_4\}$ if $\omega_3, \omega_4 \in x$ and $\omega_2 \notin x$, and $I(\{\omega_1\}) = \{\omega_1\}$. Firm 1 does not deviate at ω_1 by revealing $x \ni \omega_2$ if $u_1^N(\theta_1^L) \geq u_1(\frac{-\theta_1^L - q_2^N(\theta_1^H)}{2}, q_2^N(\theta_1^H), \omega_1)$ which was verified in the Proof of Proposition 1; firm 1 does not deviate by revealing x such that $\omega_3, \omega_4 \in x$ and $\omega_2 \notin x$ because $u_1^N(\theta_1^L) \geq u_1^B(\omega_1)$. At ω_2 firm 1 cannot modify the information structure, and thus it does not deviate. At ω_3 (or at ω_4), if firm 1 deviates by revealing $x \not\ni \omega_2$, the information structure is not modified. If it deviates by revealing $x \ni \omega_2$ its expected utility becomes $q_1(-\theta_1^M - q_1 - q_2^N(\theta_1^H))$, where $q_1 = \frac{(-\theta_1^M - q_2^N(\theta_1^H))}{2}$ is the best response of firm 1 against $q_2^N(\theta_1^H)$, i.e., its expected utility becomes $(\frac{2+\theta_1^H+3\theta_1^M}{6})^2$, which is smaller than its expected utility $u_1^B(\omega_3)$ when firm 1 does not deviate. To show that all equilibria are perfectly revealing, let $c_1(\omega_1) = c_1(\omega_2)$ or (and) $c_1(\omega_1) = c_1(\omega_3)$. In this case it is easy to check that firm 1 deviates by revealing $\{\omega_1\}$ (the argument is the same as the one given in the Proof of Proposition 1).

2. The fact that no revelation at all is not an equilibrium is obtained as before. To show that the perfectly revealing communication strategy $c(\omega_1) = \{\omega_1\}$, $c(\omega_2) = \{\omega_2\}$, $c(\omega_3) = c(\omega_4) = \Omega$ does not constitute an equilibrium, let ω_2 be the state of the world (i.e., firm 1 has a high cost and knows it). By revealing its cost its payoff is $u_1^N(\theta_1^H) = (\frac{1+2\theta_1^H}{3})^2$. If firm 1 deviates, i.e., reveals Ω, then firm 2 knows (wrongly) $\{\omega_3, \omega_4\}$ by Bayesian updating, and thus firm 2 plays $q_2^B(\omega_3)$. Playing rationally against $q_2^B(\omega_3)$, firm 1 payoff becomes $u_1(\frac{-\theta_1^H - q_2^B(\omega_3)}{2}, q_2^B(\omega_3), \omega_2) = (\frac{2+\theta_1^L+3\theta_1^H}{6})^2 > (\frac{1+2\theta_1^H}{3})^2$ since $-1/2 \geq \theta_1^H > \theta_1^L$. Therefore, firm 1 deviates from full revelation, and the perfectly revealing communication strategy is not an equilibrium. It remains to check that if firm 1 uses the communication strategy c satisfying $c(\omega_1) = \{\omega_1\}$ and $c(\omega) = \Omega$ for all $\omega \neq \omega_1$, then it has no incentive to deviate. To this aim, we first have to determine the Bayesian-Nash equilibrium when such a communication strategy is used. Of course, at ω_1, payoffs are the same as in the Nash equilibrium. Hence,

we have to determine the outputs $q_1^P(\omega_2)$, $q_1^P(\omega_3) = q_1^P(\omega_4)$ and $q_2^P(\omega)$, $\omega \neq \omega_1$, when Ω has been revealed, i.e., when firm 2's conjecture is $I(\Omega) = \{\omega_2, \omega_3, \omega_4\}$. Best responses of firm 1 are the same as in the Bayesian game without communication since its information is the same (Equations (2) and (3)). Firm 2's best response is

$$\begin{aligned}
&\arg\max_{q_2} \{(1-\beta)\gamma q_2(1-q_1(\omega_2)-q_2) + (1-\gamma)q_2(1-q_1(\omega_3)-q_2)\} \\
=&\frac{-1+\gamma(\beta+(1-\beta)q_1(\omega_2)) + (1-\gamma)q_1(\omega_3)}{2(\beta\gamma-1)}.
\end{aligned}$$

Solving this equation for q_2 using Equations (2) and (3) we get the following equilibrium output for firm 2:

$$q_2^P(\omega) = \frac{2}{3} + \frac{(1-\beta)\theta_1^H + \beta(1-\gamma)\theta_1^L}{3(1-\beta\gamma)},\ \omega \neq \omega_1.$$

It is easy to verify (see Equation (1)) that the following inequalities hold for all admissible parameters:

$$q_2^N(\theta_1^L) \leq q_2^P(\omega) \leq q_2^N(\theta_1^H),\ \omega \neq \omega_1.$$

Substituting this equilibrium value into Equations (2) and (3) we get the following equilibrium outputs for firm 1:

$$\begin{aligned}
q_1^P(\omega_2) &= -\frac{1}{3} + \frac{(\beta(1+3\gamma)-4)\theta_1^H - \beta(1-\gamma)\theta_1^L}{6(1-\beta\gamma)}, \\
q_1^P(\omega_3) = q_1^P(\omega_4) &= -\frac{1}{3} + \frac{(1-\beta)(3\gamma\beta-4)\theta_1^H + \beta(\gamma(1+3\beta)-4)\theta_1^L}{6(1-\beta\gamma)},
\end{aligned}$$

In this equilibrium, firm 1's utility is

$$\begin{aligned}
u_1^P(\omega_2) &= (q_1^P(\omega_2))^2 \\
u_1^P(\omega_3) = u_1^P(\omega_4) &= (q_1^P(\omega_3))^2,
\end{aligned}$$

and, of course, $u_1^P(\omega_1) = u_1^N(\theta_1^L)$. We have to verify that firm 1 does not deviate at ω_1 by revealing nothing (i.e., Ω) and does not deviate at ω_2 by revealing that its cost is high (i.e., $\{\omega_2\}$). If firm 1 deviates at ω_1, then firm 2 produces q_2^P at ω_1 (because

$I(\Omega) = \{\omega_2, \omega_3, \omega_4\}$). Since $q_2^P \geq q_2^N(\theta_1^L)$, the optimal production of firm 1 decreases (q_1 is decreasing in q_2 from the best reply given in Equation (2)). Hence, firm 1's utility also decreases. If firm 1 deviates at ω_2, then firm 2 produces $q_1^N(\theta_2^H) \geq q_2^P$, and thus firm 1's utility also decreases for the same reason.

3. The proof is similar to the one of Proposition 1: with high cost firm 1 has no incentive to reveal its information, and when firm 1 does not know its cost firm 1 is indifferent between revealing or not that it knows its cost. In any case, the cost is not revealed and firm 2 keeps its prior beliefs. □

Proof of Proposition 3. Property 1 is obvious from the preceding analysis. To get property 2 it suffices to remark that $q_2^P(\omega)$, with $\omega \neq \omega_1$, is increasing in γ, and thus $q_1^P(\omega_2)$ and $q_1^P(\omega_3) = q_1^P(\omega_4)$ are decreasing in γ. □

References

Clarke, R. (1983): "Collusion and the Incentives for Information Sharing," *Bell Journal of Economics*, 14, 383–394.

Gal-Or, E. (1985): "Information Sharing in Oligopoly," *Econometrica*, 53, 329–343.

Glazer, J. and A. Rubinstein (2001): "Debates and Decisions: On a Rationale of Argumentation Rules," *Games and Economic Behavior*, 36, 158–173.

Grossman, S. J. (1981): "The Informational Role of Warranties and Private Disclosure about Product Quality," *Journal of Law and Economics*, 24, 461–483.

Lipman, B. L. and D. Seppi (1995): "Robust Inference in Communication Games with Partial Provability," *Journal of Economic Theory*, 66, 370–405.

Malueg, D. A. and S. O. Tsutsui (1996): "Duopoly Information Exchange: The Case of Unknown Slope," *Internationnal Journal of Industrial Organization*, 14, 119–136.

——— (1998): "Distributional Assumptions in the Theory of Oligopoly Information Exchange," *Internationnal Journal of Industrial Organization*, 16, 785–797.

Milgrom, P. (1981): "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, 12, 380–391.

Novshek, W. and H. Sonnenschein (1982): "Fulfilled Expectations: Cournot Duopoly with Information Acquisition and Release," *Bell Journal of Economics*, 13, 214–218.

Okuno-Fujiwara, A., M. Postlewaite, and K. Suzumura (1990): "Strategic Information Revelation," *Review of Economic Studies*, 57, 25–47.

Shapiro, C. (1986): "Exchange of Cost Information in Oligopoly," *Review of Economic Studies*, 53, 433–446.

Shin, H. S. (1994): "The Burden of Proof in a Game of Persuasion," *Journal of Economic Theory*, 64, 253–264.

Vives, X. (1984): "Duopoly Information Equilibrium: Cournot and Bertrand," *Journal of Economic Theory*, 34, 71–94.